Final Report

Analysis and Optimization of Eco-Marathon Vehicle Vera from Dynamics point of view

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Acknowledgement

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Finally, the team would like to thank Christoffer Landström and David Söderblom for their help regarding aerodynamic properties of Vera as well as vortex generation theories.
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Introduction

The Shell Eco-marathon is a challenge for students around the world aiming to compete with vehicles designed to travel as far as possible using less energy. In the last Eco-Marathon competition, Chalmers vehicle Vera was still far from the winners in spite of a satisfying race with breaking the Swedish record. Therefore, there is still room for optimization and a new goal for Vera has been fixed to go beyond 2010km/litre.

In order to help the Chalmers Eco-marathon team to reach this goal in 2010, students attending the Automotive Engineering Project have been assigned to analyze and to optimize Vera. Two different teams have been formed, an engine and a vehicle dynamics teams. The goal statement having been defined by the vehicle dynamics team is the following:

“Our objective is to optimize Vera from Vehicle Dynamics point of view by analyzing the different losses, coming up with design improvements, working as a team and doing it within the next 14 weeks to help Eco-Marathon team to achieve its goals”

Vehicle Dynamics Team

In the first part, the planning followed by the team will be quickly presented as well as the risk analysis. Then, a short benchmarking on the French winning team of last year will be performed. In a following part, the losses due to different components will be analyzed by using tests, literature and models. The construction of the 2D vehicle dynamics models of Vera will then be depicted as well as the optimization of the running strategy. Finally, some design improvements will be suggested for some components.
A. Planning presentation

At the beginning of the project a planning has to be defined in order to manage the time available to reach the aim of this project. It helps to define a method of work and some limits not to be crossed to avoid waste of time and capacity of work. The following paragraph presents the all planning of the project.

1) Method and description of the project

For each project, there are players involved in a different way at different stages. In this case, three different types of players can be distinguished: customers, sponsors or stakeholders and team members.

Customers are the Chalmers Eco-marathon team and Chalmers University, sponsors are the project supervisors (Björn Pålsson, Sven Andersson and Malin Kjellberg) and the team members are the engine team and the vehicle dynamics team.

To help the Eco-Marathon team achieve its goals, some main goals have been defined. First data about the driveline losses of Vera needed to be collected, then some technical improvements have been proposed and finally the dynamic model of Vera has been rebuilt by using the new data about losses in the Matlab code. In addition, a new driving strategy has been elaborated.

In order for the sponsor to follow the work done by the team on the project, some intermediate deliverables have been defined and handed out. The first main goal was to collect enough data to carry on with the project and a document summarizing all the data that could be delivered to the customer. To collect those data, some literature and a previous implemented Matlab code were available. Moreover, some tests have been performed to measure missing data in particular about losses in the driveline of Vera. These data have been gathered and listed in a delivered sheet (“Vera data.xls” seen in Appendix 7) and a draft report has been delivered summarizing the test rigs description and the analysis of the distribution of losses along the driveline.

By analyzing the loss distribution, the second main goal was to come up with some design improvements where the main losses appeared. Some collection of data has been made about possible improvements and some tests were performed to validate the solutions.

The last goal was to work on the Vera model by implementing it with the different losses. The main output for this model is an optimal driving strategy for Vera. In addition to the new Matlab code of the Vera model, a theoretical and describing report is available.

The last deliverables are this actual final report that gathers and summarizes all the previous deliverables and the final presentation. The planning of the semester can be found in appendix 1.

2) Boundaries of the project

In order not to step out of the task, some boundaries have been defined. These boundaries established some limits of relevant things that should have been covered or not. The first boundary is that the vehicle dynamic team is limited to the clutch following the driveline. After this point, it is the engine team which was involved. Another boundary is that some specific parts on Vera cannot be changed for the meanwhile. For instance, the aerodynamic body shape could not be improved or changed this year. However, some design improvements
have been suggested. Furthermore, as students, the resources available for this project were not unlimited. Some tests have not been performed because of a lack of testing facilities. A cost study has been considered but no such problem has been encountered. Last but not least, the Shell Eco-marathon’s rules have been respected as they should even if they can be very constraining in some specific areas.

Finally, a risk analysis has been made at the beginning of the project to assess what types of events are likely to happen and slow down the group in its progress. This analysis has allowed the team to think about some preliminary solutions to minimize the impact of such an event and the project even if most of these problems have not occurred. This risk analysis is summarized in appendix 2.
B. Benchmarking: Overview of the winner vehicles of 2009 Shell Eco-marathon

This part consists simply in a short benchmarking done on French websites regarding vehicles of the last year winning team Microjoule. However, it has to be noted that because of confidential reasons, information presented here could be seen as not so useful for the following.

**Winner team:** LPTI St Joseph-La Joliverie from Nantes (France)

Prototype / Gasoline (Petrol SP 95)
Team Name: MICROJOULE
Vehicle Name: MICROJOULE
Last year best run= 3771km/L
3rd rank in 2009 Shell Eco-Marathon

The Microjoule project was born in January 1985 with the first Eco-Marathon race in France. At the very beginning, this vehicle was called “le petit Joule” (i.e the small Joule) and was made of exotic wood. It had been capable of covering 400 km with a single litre of fuel at the average speed of 30 km/h. This car was the result of the imagination of future engineers specialized in piston engine, combustion, mechanics, thermodynamics, friction reduction, testing techniques and measurement systems. The prototype vehicles were developed by mostly analyzing friction optimization in the driveline and combustion efficiency.

Regarding the last year vehicle body, the prototype geometrical characteristics are the following: weight of 30 kg (23 kg without powertrain), total length of 3 m, maximal width of 0.7m and maximal height of 0.5 m.

Microjoule is equipped with a very advanced aerodynamic carbon chassis and a carbon fibre body. As expected, only a small driver can fit inside. The drag coefficient is only 0.11. Likewise, the tires are very advanced radial ones and made by Michelin especially for the competition. Very fragile, they were engineered to have a very low rolling resistance and their lifetime is only 30 km long. The wheels are made of carbon.

The main characteristics of the engine are the following:
- Gasoline one cylinder 4-stroke engine
- Engine size of about 30.5 cm³
- Two camshafts
- Double ignition
- Injection and ignition controlled by an ECU

This type of internal combustion engine has a very specific architecture. All the technical choices are supposed to reduce the losses in order to get the best possible efficiency. The engine is running only during 6% of the total race time with full load acceleration and sets up close to the maximum efficiency. In these specific operating conditions, the engine is running lean at relatively low temperatures. Therefore, when optimizing the set up, it is interesting to analyze the following:

- Mixture preparation and filling of the cylinder
• How combustion happens…
• Combustion stability
• Knock resistance
• Combustion analyses system
• Heat losses and knock libraries

Prototype / Hydrogen
Team Name: POLYJOULE
Vehicle Name: POLYJOULE
Last year best run= 3451 km/L
3rd rank in 2009 Shell Eco-Marathon

Polyjoule is a vehicle using a Hydrogen fuel cell (500W) and two electric engines of 300W. This propulsion mode allows having better efficiency than the most precise internal combustion engines. Actually, an internal combustion engine running at the Shell Eco-Marathon is running close to 40% efficiency when the hydrogen fuel cell is running closer to 50%.

The body is similar to the one of Microjoule but with a total weight of 43 kg. This vehicle has participated 3 times at the Shell Eco-Marathon Europe in the category « hydrogen fuel cell »: in 2006, it obtained the 2nd place covering 2630 km with the equivalent of 1 litre of gasoline; it won the competition in 2007 covering 2797 km and in 2008, it finished 2nd with 2830 km/L.
C. Collecting Vera data

The first objective of the project was to establish a diagram of the different components losses along the driveline. This has been realized using testing, theoretical calculations and literature researches. However some test rigs could not be established due to their complexity and the lack of measurement facilities. For instance, the test set up for measuring the chain losses should be available by the end of the year. Therefore, the results might be obtained too late to be included into the Matlab model in the scheduled time. Hence, an assumption will be done when implementing it in the code.

1) Center of gravity measurement

Before starting any kind of tests, the center of gravity position was needed. For instance, it will be explained later on that a pendulum test has been manufactured to measure the tire rolling resistance. Since the rolling resistance depends on the normal load, the pendulum had to carry a similar front axle load so the measurements will be as accurate as possible. In the first part, the theoretical background and the test protocol will be explained. The results will then be presented.

a) Measurement protocol

The following parameters are part of the nomenclature used in this part.

\[\begin{align*}
  l_f & : \text{Distance from CG to centre of front wheel [mm]} \\
  l_r & : \text{Distance from CG to centre of rear wheel [mm]} \\
  L & : \text{Wheelbase of the vehicle [mm]} \\
  h & : \text{Distance from CG to ground when the vehicle is standing on flat surface [mm]} \\
  m & : \text{Laden mass of the vehicle [kg]} \\
  W & : \text{Laden weight of the vehicle} = mg = W_f + W_r = W_{f^*} + W_{r^*} [N] \\
  R & : \text{Laden rolling radius of wheels (Assumed to be the same for all wheels) [mm]} \\
  W_f & : \text{Front axle’s vertical load when the vehicle is standing on flat surface [N]} \\
  W_{f^*} & : \text{Rear axle’s vertical load when the vehicle is standing on flat surface [N]} \\
  W_{f^*} & : \text{Front axle’s vertical load when the front axle is raised [N]} \\
  W_{r^*} & : \text{Rear axle’s vertical load when the front axle is raised [N]} \\
  p & : \text{Vertical distance between the centres of front and rear wheels when front axle is raised [mm]} \\
  \theta & : \text{Pitch angle of the body when the front axle is raised [rad]} 
\end{align*}\]

The theoretical background of this measurement uses two vehicle conditions. In the first case, the vehicle will be on a flat surface and in the second, the front axle will be raised using blocks so that the centre of gravity height can be found. It has to be noted that the following explained measurements were taken when the shell was uninstalled. A similar test for the shell was performed to find CG location along the longitudinal axis of shell, but finding CG height was not so easy, therefore it was assumed to coincide with CG height without the shell (Therefore no effect on CG height of whole Vera).

When a vehicle is on the flat surface, writing static equilibrium equation yields:
When the front axle is raised so that the vertical distance between the centres of front and rear wheels is \( p \) (all wheels are on a flat surface so that brakes are not needed to be applied), writing moment equation around contact point of rear wheel with the ground yields:

\[
W_f^* L \cos \theta = W (l_r \cos \theta + R \sin \theta) - h \sin \theta
\]

\[
h = W (l_r \cot \theta + R) - W_f^* L \cot \theta
\]

Where \( \theta = \arcsin \left( \frac{p}{l_r} \right) \)

The following drawings illustrate the two cases of the test set up.

Using these equations above and the test protocol depicted step by step below, all the needed geometrical data can be obtained.

1. Tighten the front left brake disc’s screws. Those screws had been loosened to measure relevant dimensions of the wheel easily before the experiment was conducted. They had to be in correct position to allow the wheel to rotate.
2. Inflate the tyres: Tyres were inflated with a pressure of 5 bars to correspond to race conditions.

3. Place the seat and firewall inside the vehicle and install them correctly. These parts had been dismantled from the vehicle to be able to observe the mechanical layout of Vera before the experiment was conducted. They had to be in the vehicle to place the driver inside.

4. Check the condition of brakes for safety reasons even though they will not be used during the test. Also check if the brake pads touch the discs when brakes are not applied.

5. Place the driver inside Vera properly together with all her equipments. During the test, Shasha Xie sat in the vehicle. She was wearing her helmet. Moreover, fire extinguisher was placed in the vehicle.

6. Measure the wheelbase and laden wheel rolling radius. Wheelbase was measured, but laden wheel rolling radius was not measured due to the lack of precision of the measurement device used, instead 1.5 mm vertical deformation after loading was assumed.

7. Calibrate the scales, check their accuracy. Unfortunately, the team had no calibration mass or any other opportunity to check accuracy of the scales. Furthermore, the scales were not designed to be calibrated even though they definitely need to be calibrated after certain number of use.

8. Measure axle loads when the vehicle (driver inside) is on a flat surface. Never apply the brakes unless driver feels that the car is sliding off the scales uncontrollably. If this happens, the test should be restarted. Wheels should rotate freely; otherwise an additional moment caused by residual tyre deformation will affect the equilibrium equations.

9. Find/build a step to place the front axle on. In the experiment three blocks whose heights were 100 mm were placed on top of each other.

10. Check calibration and accuracy of scales again. Not performed because of reasons explained in 7.

11. After placing the front axle of Vera on the step, measure axle loads: Since the height of scales were the same, “p” remained the same = 300mm. Never apply the brakes unless driver feels that the car is sliding off the scales uncontrollably. If this happens, the test should be restarted. Wheels should rotate freely, otherwise an additional moment caused by residual tyre deformation will affect the equilibrium equations.

12. Perform preliminary calculations to see whether the measurements agree with expectations or estimations. If something wrong is suspected, repeat the steps explained above.
b) Results

As explained earlier, the vehicle dimensions have been measured. The picture below illustrates the values obtained for wheelbase, front track width and unladen wheel radius.

The car was settled on 3 scales (one for each wheel). They did not really give the same weight because of some discrepancies and they could not have been calibrated for evident reasons. The difference between them when measuring was about 0.3kg. In order to reduce these inaccuracies, the tests described above have been performed 3 to 5 times and an average value has been taken for the calculation of the centre of gravity position. The variation of mass in the tests can be due to the driver movements between each test (especially lateral load transfer at the front wheels).

The table below summarizes the masses given by each scale.

<table>
<thead>
<tr>
<th>Test</th>
<th>Front left</th>
<th>Front right</th>
<th>Rear</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.8</td>
<td>24.4</td>
<td>25.2</td>
</tr>
<tr>
<td>2</td>
<td>24.1</td>
<td>25.0</td>
<td>25.5</td>
</tr>
<tr>
<td>3</td>
<td>24.1</td>
<td>25.0</td>
<td>25.5</td>
</tr>
<tr>
<td>Average</td>
<td><strong>24.3</strong></td>
<td><strong>24.8</strong></td>
<td><strong>25.4</strong></td>
</tr>
</tbody>
</table>

For the second part of the test, the team had to raise the front of the car. We tried with a height for the support of 200 mm and the results are presented below. The difference between the
weight results with a slope and without was not significant enough. Therefore the height was increased up to 300 mm. The results are depicted in the following table.

<table>
<thead>
<tr>
<th>Test</th>
<th>Front left</th>
<th>Front right</th>
<th>Rear</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.7</td>
<td>24.0</td>
<td>25.2</td>
</tr>
<tr>
<td>2</td>
<td>24.3</td>
<td>24.1</td>
<td>25.5</td>
</tr>
<tr>
<td>3</td>
<td>24.3</td>
<td>24.1</td>
<td>25.5</td>
</tr>
<tr>
<td>Average</td>
<td><strong>24.4</strong></td>
<td><strong>24.1</strong></td>
<td><strong>25.4</strong></td>
</tr>
</tbody>
</table>

From these results the position of the centre of gravity can be calculated. The centre of gravity found is very low, that is why the variation of weight with and without a slope was not so important.

To summarize, all the dimensions measured during this test are presented below.

![Figure 3: Position of the center of gravity](image)

Average of measured values and calculated values without the shell:

When taking the shell into account the height of the centre of gravity will be assumed to remain the same.

Estimated values with the shell:

The weight of the shell has been measured using two simple scales. Approximately 4kg on the front axle and 2kg on the rear has been measured. The mass distribution is therefore around two thirds on the front axle.

<table>
<thead>
<tr>
<th>Measured values</th>
<th>Without the shell</th>
<th>Estimated values with the shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_f$</td>
<td>482 N</td>
<td>$482 + 40 = 522$ N</td>
</tr>
<tr>
<td>$W_r$</td>
<td>249 N</td>
<td>$249 + 20 = 269$ N</td>
</tr>
<tr>
<td>$W_f^*$</td>
<td>476 N</td>
<td></td>
</tr>
<tr>
<td>$W_r^*$</td>
<td>258 N</td>
<td></td>
</tr>
<tr>
<td>$l_f$</td>
<td>481 mm</td>
<td>481 mm</td>
</tr>
<tr>
<td>$l_r$</td>
<td>929 mm</td>
<td>929 mm</td>
</tr>
<tr>
<td>$h$</td>
<td>223 mm</td>
<td>223 mm</td>
</tr>
</tbody>
</table>

The values computed with the shell will be used in the following whenever they are needed.
2) Coast down test

This was a proposed test to see Vera’s real life performance to measure overall rolling resistance and aerodynamic drag coefficient, however it was not allowed because of the risk of damaging the vehicle.

The test is recommended by Bosch Automotive Handbook. First of all, a straight road and a test day with no wind should be chosen. Then two speeds (one is high and the other one is low) that the vehicle can reach with its powertrain should be decided. Those can be chosen as $V_{\text{start}1} = 50 \text{ km/h}$ and $V_{\text{start}2} = 15 \text{ km/h}$, for instance. Then, the speed difference at which the test is terminated should be chosen. This speed difference ($\Delta V$) should not be more than 5 km/h as a constant acceleration for this test is assumed (3 or 4 km/h can be used to perform coast down test on Vera). This means that $V_{\text{stop}1} = 47 \text{ km/h}$ and $V_{\text{stop}2} = 12 \text{ km/h}$ if a speed difference of $\Delta V = 3 \text{ km/h}$ is decided to be used. The time elapsed ($t_1$ and $t_2$) for the vehicle to coast down from the initial speeds ($V_{\text{start}1}$, $V_{\text{start}2}$) to the final speeds ($V_{\text{stop}1}$, $V_{\text{stop}2}$) for both tests are measured. Following calculations should be performed to get the dimensionless drag coefficient and rolling resistance coefficient:

$$a_1 = \frac{V_{\text{start}1} - V_{\text{stop}1}}{t_1} \text{ in } [(\text{km/h})/s]$$

$$a_2 = \frac{V_{\text{start}2} - V_{\text{stop}2}}{t_2} \text{ in } [(\text{km/h})/s]$$

$$V_{av1} = \frac{V_{\text{start}1} + V_{\text{stop}1}}{2} \text{ in } [\text{km/h}]$$

$$V_{av2} = \frac{V_{\text{start}2} + V_{\text{stop}2}}{2} \text{ in } [\text{km/h}]$$

$$C_D = \frac{6M_{\text{tot}}(a_1 - a_2)}{A(V_{av1}^2 - V_{av2}^2)}$$

$$f_r = \frac{28.2(a_2V_{av1}^2 - a_1V_{av2}^2)}{1000(V_{av1}^2 - V_{av2}^2)}$$
3) Rolling resistance

a) Fundamental calculation

The following figure is the schematic drawing of the pendulum.

![Figure 4: Free body diagram of one wheel](image)

Equations of motion for the pendulum shown above:

\[\sum F_x = m(a_x)_{CG}\]  \hspace{1cm} (Eq.1)

\[\sum F_y = m(a_y)_{CG}\]  \hspace{1cm} (Eq.2)

\[\sum M_{CG} = I_{CG}(d^2\theta)/(dt)^2\]  \hspace{1cm} (Eq.3)

The equations of motion above are straightforward. In order to be able to use them to get the differential equation of pendulum device, a kinematic analysis should be made first. By observing the system, one can write following relative acceleration expressions:

\[a_{CG} = a_c + \alpha \times r_{CG/c} + \omega \times (\omega \times r_{CG/c})\]  \hspace{1cm} (Eq.4)

Where

\[\alpha = \dot{\theta} k\]  \hspace{1cm} (Eq.5)

\[\omega = \dot{\theta} k\]  \hspace{1cm} (Eq.6)

\[r_{CG/c} = h \sin \theta i + (r - h \cos \theta) j\]  \hspace{1cm} (Eq.7)
To be able to calculate the overall centre of gravity acceleration, one should know the acceleration at point C. One assumption here is that the pendulum rolls back and forth without sliding. Therefore:

\[
\vec{a}_C = \left( \dot{\theta} \right)^2 r \hat{j}
\]

(Eq.8)

Substituting equations 5-8 into equation 4 gives the acceleration of CG at any instant of movement:

\[
\vec{a}_{CG} = \begin{bmatrix}
-\dot{h}\sin(\theta) \\
-\ddot{h}(r-h\cos(\theta)) \\
\dot{h}\sin(\dot{\theta}) + h\cos(\dot{\theta})
\end{bmatrix} \hat{j}
\]

(Eq.9)

Note that the component along x axis gives the \(a_x\) and component along y axis gives the \(a_y\).

Equation 9 is the end of kinematic analysis.

For kinetic analysis, acceleration values found from equation 9 should be substituted into equations 1 and 2:

\[
F_i = -m\theta r + mh\cos(\theta)\dot{\theta} - mh\sin(\dot{\theta})^2
\]

(Eq.10)

\[
W = mh\sin(\dot{\theta}) + mh\cos(\dot{\theta})^2 + mg
\]

(Eq.11)

The final equation comes from summing moments around CG (recall equation 3 and keep in mind that \(M_{\text{rolling}} = f \cdot r \cdot \text{r.sgn(dθ/dt)}\)):

\[
F_i(r - h\cos(\theta)) - Wh\sin(\theta) - f,Wr \cdot \text{sgn}(\dot{\theta}) = I_{CG} \ddot{\theta}
\]

(Eq.12)

Combining equations 10, 11 and 12 leads to a “nice” equation:

\[
\ddot{\theta} = -\frac{mhr \sin(\dot{\theta})^2 - mgh\sin(\theta) - f, mh\cos(\dot{\theta})^2 \cdot \text{sgn}(\dot{\theta}) - f, mhr \text{sgn}(\dot{\theta})}{I_{CG} + m(r^2 + h^2) - 2mhr \cos(\theta) + f, mhr \sin(\theta) \text{sgn}(\dot{\theta})}
\]

(Eq.13)

As seen, equation 13 cannot be solved analytically, but a numerical solution is possible. However, this approach will not be helpful in this type of experiment, since a simple relation between easily observable quantities (e.g. the time elapsed for the pendulum to swing until a complete stop) and rolling resistance is looked for. As a result, equation 13 should be linearized for small variations at around zero setpoint. Hence (overbar represents the setpoint, tilde represents a small variation):

\[
\theta = \bar{\theta} + \tilde{\theta} \rightarrow \bar{\theta} = \tilde{\theta} \rightarrow \bar{\theta} = \tilde{\theta}
\]
If there is no sharp change in the input (e.g. step input) over time, then the derivatives will remain small. This means that second or higher degree terms can be neglected. Recalling that \( \sin \theta \approx \theta \) and \( \cos \theta \approx 1 \) for small variations around zero setpoint, one can write “useful” linearized equation:

\[
\left[I_{CG} + m(r - h)^2\right] \ddot{\theta} + mgh\dot{\theta} = -f_mgr\text{sgn}\dot{\theta}
\]  
(Eq. 14)

Simultaneous simulations of equations 13 and 14 in Simulink have shown very similar results. Therefore linearization has not caused a big trouble.

The Simulink model of the tire is in appendix 4.

Since equation 14 includes signum function, the sign of the term on the right-hand side changes at the end of every half period of oscillation (this is when pendulum stops and starts to swing towards the other side so the sign of speed changes). Therefore, a solution is valid only for half a period. However, since the initial velocity is zero and it is zero again after half period, then the process continues in a way which is same as giving another initial angle to the pendulum and letting it swing. As a result, what happens on the first half-period will repeat itself in the second half-period. The attenuation of amplitude (angular position) in one period can be found as:

\[
\text{Attenuation}_{\text{halfperiod}} = \frac{2f_mgr}{mgh} = \frac{2f_r}{h}
\]

\[
\text{Attenuation}_{\text{fullperiod}} = 2\times \text{Attenuation}_{\text{halfperiod}} = \frac{4f_r}{h}
\]  
(Eq. 15)

b) Design of pendulum

The pendulum consists of three parts, two wheels, two steel plates, and two \$80 steel bars as seen in the following photo:

![Figure 5: Pendulum test for rolling resistance](image)
Wheels

The wheels are Vera’s front wheels. The wheel rim material is aluminium. The unloaded radius of the wheel is 237.5 millimetres. On one side of the wheel, there are six holes for the attached disc brake. These holes are used to attach the steel plate. The weight of one wheel is 1.175 kg. Tyre inflation pressure is about 5 bars.

Steel plate

The steel plates are designed as described below and manufactured by CNC machine. The purpose of these plates is to attach the \( \phi 80 \) bar to the wheel and align two wheels so that they are parallel to each other. The steel plate is attached to the wheel by using six bolts. The weight of one plate is about 2.65 kg. The technical drawing can be found in appendix 6.

\( \phi 80 \) steel bar

The \( \phi 80 \) steel bars are designed as given below. The purpose of the bars is to increase the normal load on the tyres and provide a changing torque to swing the wheels like pendulum. The weight of one \( \phi 80 \) steel bar is about 15.9 kg. The technical drawing can be found in appendix 6.

Centre of gravity for the pendulum and final conversion factor

Although crucial parts were designed and drawn in solid modelling software; some parts such as nuts, bolts and washers were missing in the software. Since manual calculation of centre of gravity position for the assembly is not difficult, this is preferred. Since the pendulum is symmetrical, calculating the distance “h” for one side (left or right) will be enough. While doing so, half mass of the \( \phi 80 \) bars whose CG locations are on the geometrical centre of their circular cross section, should be considered in the calculations. CG positions of the wheels are assumed to be coinciding with their rotational centres. Masses of nuts, bolts and washers are neglected. CG positions of the steel plates are about 85 mm lower than the rotational centre of wheels. Summing “mass moments” around CG (the vertical distance between the centre of one wheel and the centre of gravity position for the assembly is denoted as “h”):

\[
1.175h + 2.65(h - 85) + 7.95(h - 95) - 7.95(180 - h) = 0 \rightarrow h \equiv 122\text{mm}
\]

Since 1-2 mm of tyre deflection is assumed, loaded wheel radius becomes approximately 236 mm. Substituting “h” and loaded radius values into equation 15 gives the following useful expressions (radian to degree conversion should also be performed):

\[
\text{Attenuation}_{\text{fullperiod}} = 443.34 f_r \text{[°]} \quad \text{(Eq.16)}
\]

Suppose that one tests the pendulum from 20 to 10 degree vibration amplitude. The experimenter counts 12 periods during the test. Then the average rolling resistance is \( f_r = (20-10)/(443.34*12) \approx 0.0019. \)

Finally, note that whole pendulum weighs approximately 39.5 kg.
c) **Test protocol and procedure**

1. Inflate the tyres with 5 bars; check the stiffness the pendulum system by checking the firmness of screws, bolts and nuts.

2. Find a proper, flat surface for the experiment: A surface which is similar to race track’s asphalt should be chosen to obtain proper rolling resistance values. Moreover, the surface should be flat. After some investigation with measurement tools, one of student house’s garage located close to laboratory was chosen.

3. Decide the initial angle of the pendulum: First decision was to give 10 degrees of angle to the pendulum and let it swing, since variations should be small enough for equation 16 to hold. By measuring period time and the total time, one can calculate how many times it swings until a complete stop. Then, applying equation 16 would give the average rolling resistance. However, because of extremely low speeds at the end of the test (especially when the angle amplitude was less than 5°), rolling resistance went down to extremely low values which were unrealistic (Since rolling resistance is mainly caused by hysteresis of the tyres, and hysteresis of polymers strongly depends on deformation speed and amount of deformation, the results was not so surprising). In order to get realistic values, a second test with an initial angle of 20 degree was chosen and this time, the pendulum was allowed to swing until 10 degree of angular amplitude so that the speeds and deformation amounts remained at sufficient levels. Numerical simulation of exact differential equation was compared with numerical simulation of linearized differential equation to check if this relatively big initial angle would cause a big deviation from linearized differential equation. The deviation was small enough.

4. Find a way to measure initial condition of the pendulum: Because there were no angular position transducer and corresponding data acquisition system were given at group’s disposal, another solution had to be used. Since loaded wheel radius was known, it was decided to convert angular displacement to translational displacement by using \(x=\theta*(\text{loaded radius})\) relation. To be able to realize this in practise, indicators with a spacing of \(x=10*(\pi/180)*236 \approx 41\) mm were drawn on the paper.

5. Measure the number of periods that the pendulum travels: One way to do so was given in 3\(^{rd}\) step. The other option can be to count the periods manually. Since the latter technique seemed to be more reliable, the group went for this technique.

6. Repeat the tests to obtain better results in the end.

Followings are pictures from experiment:
d) Test results

As explained before, starting the test with 10 degrees and waiting for the pendulum to come to a complete stop was not a proper way of testing the rolling resistance. In this test, the average result is as follows (surface was damp), details can be found in appendix 3.

<table>
<thead>
<tr>
<th>Wet</th>
<th>Start angle</th>
<th>5°</th>
<th>8°</th>
<th>10°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period Number</td>
<td>34,25</td>
<td>37,33</td>
<td>40,50</td>
</tr>
<tr>
<td></td>
<td>Attenuation Time</td>
<td>25,75</td>
<td>28,87</td>
<td>31,70</td>
</tr>
</tbody>
</table>

By looking at the data above, one can easily calculate the rolling resistance by making use of Eq.16. However, the calculated rolling resistance values were about 0.0006, which was lower than a steel train wheel. Therefore, the test data above was unrealistic and had to be skipped.

Following gives the test results for a pendulum test started at 20 degrees of amplitude and ended at 10 degrees of amplitude:

| Asphalt | Angle from 20° to 10° | Period Number Average | 9,33 | | Attenuation Time Average | 8,00 | | Rolling Resistance Coefficient | 0,00242 |

The test results shown above reveal reasonable values for tyre rolling resistance coefficient although they are higher than expected (compared to an overall rolling resistance coefficient of 0.0024 - estimated from GPS data). This high value resulted from relatively high rms value of torque applied to the whole pendulum during the test. Since high torque means bigger longitudinal deformations, tyre rolling resistance turned out to increase. In order to test whether this high rms value of torque affected the results or not, lower bar was removed from the pendulum to reduce the torque. This means that both “h” and the weight of the pendulum were also reduced. However, this modification also reduced the rms values of travel speed and vertical load fluctuations which means that actual race conditions were not simulated properly. The average result follows (details can be found in appendix 3).

<table>
<thead>
<tr>
<th>Elapsed Time (s)</th>
<th>Number of periods</th>
<th>Rolling resistance coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.3</td>
<td>9.6</td>
<td>0.001608</td>
</tr>
</tbody>
</table>

Note that since distance “h” was changed, equation 16 was changed also. New relation was $\text{Attenuation\_fullperiod} = 651.65*f_r$ in terms of degrees. The values seemed to be more reasonable now, but one should be aware that speeds and normal load on the tyres were lower in this test which does not correspond to racing conditions. Therefore, the reliability of this test was not so good either. One solution for this problem can be to take the average of rolling resistance coefficients as a better value ($f_r$ around 0.0019).

One final note is that the test tyres were not in a good shape which might be a reason for higher rolling resistance coefficients.
4) Wheels’ inertia measurements

The aim to determine the wheel inertia is to measure afterwards the one-way clutch losses on the rear wheel. One cannot measure directly the rear wheel inertia because of a half part of the clutch fixed on it. Then it was assumed that all the front and rear wheels of Vera have the same inertia.

First of all, the front wheel inertia has been measured and then the importance in the inertia approximation of the one-way clutch half part fixed on the rear wheel has been estimated.

a) Protocol

In order to know the inertia of the wheel, one can use the test technique of the mass based on the equation \( \sum \tau = \bar{T}\dot{\theta} \). A mass is attached on the wheel with a rope. It applies a torque on the steady wheel and makes it rotate. The weight and the distance the mass will run are known so that one can deduce the torque applied and also its speed and acceleration and then deduce the acceleration on the wheel.

The following figure is a sketch of the test set-up. There are four positions on the wheel that correspond to the attachment of the rope. Four tests are needed because of the unbalance of the wheel.

![Figure 7: Sketch of the wheel inertia test set-up](image)

The following nomenclature will be the one used in this part:

- \( S \): dropping distance [m]
- \( T \): dropping time [s]
- \( A \): linear acceleration [m/s\(^2\)]
- \( \dot{\theta} \): angular acceleration [rad/s\(^2\)]
- \( M \): mass [Kg]
- \( G \): gravity [m/s\(^2\)]
- \( T \): string tension [N]
- \( R \): radius of the wheel [m]
- \( \tau \): Torque from string tension [N.m]
- \( \bar{I} \): Mass Moment of Inertia [Kg.m\(^2\)]
**Test Procedure**

1. The wheel is settled on its axle. The screw that prevents the wheel from translating along the axle shall not be tightened because it will add a frictional moment (axial load on the bearing).

2. Attach the 45 grams weight to the wheel by using string and tape. The string has to be thin so that it will not slide off the tyre.

3. Set the weight up to 1 meter height by spinning the wheel. It is better that the string goes around the wheel in order to compensate its unbalancing. Therefore, four different positions will be used.

4. Release the weight and measure the dropping time.

5. Repeat No. 2-3 two more times.

6. Move the attachment point of the string and wheel 90 degree clockwise to avoid the unbalanced wheel problem.

7. Repeat the test again from step No 2-4.

8. Do the same for 180° and 270° positions as in the schematic picture above.


10. Change the mass weight to 75, 100, 130 grams and increase the height to 1.1 meter to reduce the time measurement error.

11. Repeat the test again from No.3-8.

**Equations**

Linear and Circular Equations of Motions:

\[ a = \frac{2\times S}{t} \quad \text{and} \quad \ddot{\theta} = \frac{a}{r} \]

Equations of the inertia test applied to the mass (free body):

\[ \sum F = ma \]
\[ mg - T = ma \]
\[ T = m(g - a) \]

Applied to the wheel:

\[ \sum \tau = \ddot{\theta} \]
\[ T \times r = I \ddot{\theta} \]
\[ m(g - a) \times r = I \dddot{\theta} \]
\[ m(g - a) \times r = I \frac{a}{r} \]
\[ I = m \left( \frac{g}{a} - 1 \right) \times r^2 \]
b) Front Wheel Results

After calculating the inertia for each test, the statistic method has been used with 95% confidence interval (T-tables) to find the average value and reliability of the test. Results of the test post-processing can be found in appendix 5.

The following graph shows the front wheel inertia average value in four different weights tests with the maximum and minimum values. Finally, the front wheel inertia has been found to be about 0.04036 Kg.m².

![Figure 8: Front wheel inertia confidence intervals](image)

### Wheel inertia mean values and confidence interval

- Inertia: $kg.m^2$
- Weight: kg

![Figure 9: Clutch part fixed to the rear wheel](image)

c) Rear Wheel

Having performed satisfying measurements of the inertia of the front wheel, it was assumed that the rear wheel should have the same inertia. However, it has to be verified whether the part of the one-way clutch fixed to the wheel is playing a role or is to be neglected. To date, the front wheel inertia obtained from the test is 0.04036 kg.m².

In order to estimate the inertia of the part of the one-way clutch without performing the complete test procedure to get the total inertia of the rear wheel, Catia V5R16 has been used. After measuring the necessary dimensions as well as the weight, the part has been designed in Catia as seen on the screenshot below.
Since the material used to manufacture this part was not known, the weight and the calculated volume have been used to get the density. Then, by simply using Catia functions, the inertia has been calculated and a value of 0.00003 kg.m² has been found.

As expected, this result was negligible compared to the inertia of the wheel. However, it will still be taken into account by adding it to the assumed rear wheel inertia. The final results of wheels inertia (front and rear) are summarized in the table below:

<table>
<thead>
<tr>
<th>Inertia of the front wheel</th>
<th>0.04036 kg.m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia of the rear wheel</td>
<td>0.04039 kg.m²</td>
</tr>
</tbody>
</table>

5) Bearing losses

Due to the complexity and time available for setting up a test for bearing losses, another mean of determining the loss had to be found. The bearings used on Vera’s wheels are single row groove ball bearings manufactured by SKF. Their reference number is 61802-2z and all characteristics can be found on SKF website. From all the data available on SKF, the moment of friction due to bearings has been calculated in a first part. The second part will be suggesting a test rig that could be performed in the future to evaluate the losses coming from bearings.

a) Power loss calculations

In order to estimate the frictional moment, the SKF model will be used. This model is only valid under certain conditions which are that the bearing should operate at normal operating conditions with a good lubrication (grease in this specific case). The model used states that:

\[ M = 0.5\mu Pd \]

Where:

- \( M \) = total frictional moment [N.mm]
- \( \mu \) = constant coefficient of friction for bearing (for a groove ball bearing \( \mu = 0.0015 \))
- \( P \) = equivalent dynamic bearing load [N]
- \( D \) = bearing bore diameter [mm]

Just for information, the total frictional moment model takes into account the sum of all resistance moments (rolling frictional, sliding frictional, frictional moment of the seal and frictional moment of drag losses, churning, splashing...).

Hence, several parameters were to assume to be used in the model. These assumptions have been mainly made to be coherent with race conditions.

The first assumption was about the rotational speed of the bearing. During the race, the vehicle is supposed to be driven at an average speed of 30 km/h (basically, the strategy is to switch the engine on and accelerate up to 40 km/h and then let the vehicle speed decreases to
Therefore, using the laden radius of the wheel, the rotational speed has been calculated.

Due to the strategy employed during the race, the bearing losses will be calculated for each speed from 20 to 40 km/h.

To estimate the viscosity of the grease, a second assumption was needed. Actually, the viscosity depends on the rotational speed and the operating temperature. Since the race takes place during warm seasons, the operating temperature has been set to 25°C.

The curve below shows how the frictional moment of the bearings is dependent on the speed of the wheels, assuming that the axial load is equal to zero as the vehicle is unloaded.

![Figure 10: Frictional bearing moment versus speed](image1)

From the graph above, it can be said that the frictional bearing moment in almost constant when varying the speed. As one should stick to the race conditions, laden vehicle with axial loads has to be studied. The next figure shows the frictional bearing moment depending on the axial and radial loads at an average speed of 30 km/h. The values have been obtained from the SKF website. The ranges of axial and radial loads have been evaluated regarding the mass transfer and the acceleration due to the speed in cornering. As one can see on the map below, loads play an important role in the frictional moment.

![Figure 11: Frictional bearing moment depending on axial and radial loads](image2)
b) Suggested test

Before computing the frictional moment on the SKF website as explained earlier, some thinking had been done in order to suggest a test set up to measure bearing losses. The sketch below illustrates the test set up that was retained. It consists in mounting the bearings on a main rotating shaft very tightly and a free pipe on the outer side of the bearings. This free pipe will be able to move if a moment is applied on it. A mass will be added fixed to this pipe as on the right side of the picture and by measuring the angle (or the distance), the frictional moment coming from the bearings can be calculated.

Figure 12: Suggested bearing losses test set-up

However, some aspects of this test set up have still to be thought of as for instance how the bearings can be fit, how to measure the angle, manufacturing processes... Moreover, one cannot be sure that the results obtained during such testing would be more accurate than ones from SKF model presented in the previous part (e.g. by using this method, effect of axial load on the resistance moment cannot be evaluated simply). Therefore, the data obtained from SKF data will be used when building the dynamics model of Vera in Matlab.

6) One-way clutch and rotational wheel aerodynamics drag

a) Test protocol

This test is about investigating the loss in one-way clutch (OWC) of the rear wheel axle. To achieve that, two tests (without, and with OWC) have been decided with two different methods of analysing (Conservation of energy, and Newton’s law of motion) for comparison purpose. In these tests, the rear wheel travels from 40 to 20 km/h, same as the competition driving strategy. To perform the tests, the wheel will be unloaded as explained earlier and will be rotated by some compressed air. Moreover, a speed sensor is used to record the time and speed while the wheel decelerates. For the first test, the rear wheel without the jaws of the OWC will be tested. This test will give the bearing, aerodynamic energy and moment losses by using method 1 and method 2 explained in the following. Moreover, since the aerodynamic constants of both tests should be equal, the rotational aerodynamic drag from the wheels can be estimated as well. In the second test, the jaws of the OWC will be installed to the rear axle.
This test will give the OWC, bearing, and aerodynamic energy losses by using method 1 and method 2.

In a first part, the theory used to perform the tests and deduce the one-way clutch losses will be depicted. Afterwards, the test procedure will be explained and the results obtained so far will be presented. However, it has to be noted that the post-processing is not finished for the second method.

In this part, the following nomenclature will be used:

\( \bar{I} \) Mass Moment of Inertia [Kg.m\(^2\)]

B Bearing loss moment [N.m]

D Drag force [N]

C Aerodynamic constant

R Radius [m]

\( v \) Linear speed [m/s]

\( \omega \) Rotational speed [rad/s]

\( \dot{\omega} \) Rotational acceleration [rad/s\(^2\)]

OWC one-way clutch loss work [kg.m\(^2\)/s\(^2\)]

**Method 1 – Law of Conservation of Energy**

By using the equation of energy conservation, the calculation below can be stated.

Rotational Energy = \( \frac{1}{2} \bar{I} \omega^2 \)

Bearing energy loss = \( \int_{t_1}^{t_2} P \, dt = \int_{t_1}^{t_2} B \omega \, dt \)

Aerodynamic energy loss = \( \int_{t_1}^{t_2} P \, dt = \int_{t_1}^{t_2} D \omega \, dt = C \omega^2 \)

\( = C \int_{t_1}^{t_2} \omega^3 \, dt \)

**Test 1 without OWC**

Inertial energy@40Km/hr = Inertial rotational energy

Final energy@20Km/hr = Final rotational energy + Bearing energy loss + Aerodynamic energy loss

From the law of energy conservation, one can get:

Inertial energy = Final energy

Inertial rotational energy = Final rotational energy + Bearing energy loss + Aerodynamic energy loss

\[ \frac{1}{2} \bar{I} \omega_1^2 = \int_{t_1}^{t_2} B \omega \, dt + C \int_{t_1}^{t_2} \omega^3 \, dt + \frac{1}{2} \bar{I} \omega_2^2 \]

\[ C = \frac{\frac{1}{2} \bar{I} \omega_1^2 - \frac{1}{2} \bar{I} \omega_2^2 - \int_{t_1}^{t_2} B \omega \, dt}{\int_{t_1}^{t_2} \omega^3 \, dt} \quad [Eq.1] \]
This test provides the aerodynamic constant C that will be the same for both tests, with and without the jaws of the OWC. Hence, using this constant in the second test post-process, one can get the OWC losses.

Test2 with OWC

Inertial energy@40Km/hr = Inertial rotational energy

Final energy@20Km/hr = Final rotational energy + Bearing energy loss + Aerodynamic energy loss + OWC energy loss

From the law of energy conservation

Inertial Energy = Final Energy

Inertial rotational energy = Final rotational energy + Bearing energy loss + Aerodynamic energy loss + OWC energy loss

\[ \frac{1}{2} I \omega_1^2 = \int_{t_1}^{t_2} B \omega dt + C \int_{t_1}^{t_2} \omega^3 dt + \frac{1}{2} I \omega_2^2 + OWC \]

\[ OWC = \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2 - \int_{t_1}^{t_2} B \omega dt - C \int_{t_1}^{t_2} \omega^3 dt \]  

[Eq.2]

In the right part of this equation, each term is known assuming that C stays unchanged from the test without the one-way clutch. The losses coming from the clutch can therefore be computed.

**Method2 – Newton’s laws of motion**

Test1 without OWC

\[ \tau = I \omega \]
\[ B + D \times R = \bar{I} \omega \]
\[ B + C \omega^2 = \bar{I} \dot{\omega} \]  

[Eq.3]

Plot \( \dot{\omega} \) against \( \omega \)

Test2 with OWC

\[ \tau = \bar{I} \omega \]
\[ B + D \times R + OWC = \bar{I} \omega \]
\[ B + OWC + C \omega^2 = \bar{I} \dot{\omega} \]  

[Eq.4]

Plot \( \dot{\omega} \) against \( \omega \)

(4) - (3) = OWC loss moment

Test1:

1. Disassembling the rear wheel in order to remove the jaw in the one-way clutch
2. Reassembling the rear wheel
3. Spinning the wheel up to 40 km/h by using a compressed air pistol
4. Measuring the time and speed during the wheel decelerate from 40 to 20 km/h by using the bicycle computer
5. Repeating the test 2 more times to get a better accuracy
6. Post processing the data by using method 1 and method 2.
Test2

1. Disassembling the rear wheel to reinstall the jaw back in position
2. Reassembling the rear axle
3. Spinning the wheel up to 40 km/h by using a compressed air pistol
4. Measuring the time and speed during the wheel decelerate from 40 to 20 km/h by using the bicycle computer
5. Repeating the test 2 more times
6. Post processing the data by using method 1 and method 2.

b) Post-processing

The bicycle computer gives some data in excel form. To facilitate the post-processing, time and speed data were transferred into a Matlab data file (.dat). The results for all the tests are illustrated in the following graph.

![Comparison of the decreasing velocity of the wheel with and without OWC](image)

As seen from the graph, all the three without one-way clutch (OWC) tests results are almost similar to each other as likely as a single curve and the same trend happens in the test with OWC. The difference is that the speed reduction from 40 to 20 km/h of the tests with OWC is faster than the one without OWC by 9.56 seconds.

*From method 1*

After testing the rear wheel without OWC, the results were analysed by using equation 1 in Matlab to find the aerodynamic constant in order to put it into equation 2 to find the OWC losses in the second test. And the results of aerodynamic constant are $1.53 \times 10^{-5}$, $1.54 \times 10^{-5}$, and $1.57 \times 10^{-5}$ for test1, test2, and test3 respectively. It can be concluded that the repeatability of the test was good enough so no additional test was needed even if the aerodynamic losses are a bit more in test 3.

Then the test with OWC has been performed and analysed by using equation 2 in Matlab and the OWC losses were found as well as the aerodynamic and bearing losses as shown in the bar chart below which illustrates the different losses during the test. However, one cannot be
satisfied with it because those values are only for this specific test. It will thereby be necessary to find a specific value for each loss for a given speed (between 20 to 40 km/h) in order to have them as an input for the vehicle dynamics model of Vera.

Figure 14: Distribution of losses on three tests

From this chart, the aerodynamic losses are very high compared to other losses and this seems to be the main loss of the rear wheel axis. The losses are 28.2515, 28.5413, and 29.2833 Joules for test 1, test 2, and test 3 respectively. Also, for bearing losses, the results are very tiny (0.7725, 0.7824, and 0.7789 Joules for three tests). Finally, the OWC losses results are 5.2505, 5.1220, and 4.2123 Joule for three tests. These results are still low when compared with the aerodynamic one but much higher than the bearing losses.

From the losses distribution chart above, the losses can be calculated in percentage as shown below.

Figure 15: Losses distribution on the rear wheel

Losses Distribution

- Aerodynamic Loss: 84%
- One-Way Clutch Loss: 14%
- Bearing Loss: 2%
The main loss of the rear wheel axis is the aerodynamic loss which is about 84 percent of total losses. And the second is the one-way clutch loss (14%). Finally, the least loss is the bearing loss, is about 2 percent.

*From method 2*

By using test results seen in figure 13 and method 2, a graph illustrating the angular speed versus mass moment of inertia times angular acceleration can be plotted. This figure can be found below. The difference between the two curves (with and without OWC) represents the one-way clutch moment of loss.

![Figure 16: Total resistance moment versus rotational speed (method 2)](image)

From the figure above, the difference between the two curves is about 0.00335 N.m. Unfortunately, the trends of two curves are quite linear in the high velocity area which is not a normal trend of aerodynamic moment loss. The rotational aerodynamic losses are supposed to be proportional to the square of the rotational velocity of the wheel. This may come from the fact that fitting curve tools have been used during the post-processing of the data. Indeed, the fitting curve tools require the data before 20 km/h and after the 40 km/h so that the beginning and ending trends will not be distorted. However, this result can still be valid since the difference between the two curves is constant and represents the value of OWC moment loss.

As a conclusion for this part one should say that both methods give close results. Using the first method for the rotational velocity of 35 rad/sec, the resistance moment of 0.00361 Nm can be obtained. Calculating the same moment using the second method, one can get 0.00335 Nm. Also, the drag caused by spoke of the wheel and the shear forces caused by surface of the tire are suspected to be the main source of rotational aerodynamic drag for the wheel.
7) Chain losses

While transferring the power using the sprocket-chain system there are always some losses in energy. One can assume that friction is the dominant loss mechanism. There are several ways of measuring the mechanical efficiency of this system. Two of them will be suggested in the following, one dynamic test and one static test. Furthermore, some research has been done without much success.

a) Test protocol suggested

**Dynamic Test**

In this test average mechanical efficiency is going to be measured. In order to calculate this efficiency, the power input to the front chain ring needs to be measured and then it should be compared to the power output on the rear drive sprocket. The ratio between the output power and the input power is used to quantify the overall efficiency of the system. To determine the powers in the drive and driven shafts the torques on each shaft should be measured. Knowing the rotation rate of the drive shaft and the gear ratio, the rotation rate of the driven shaft can be calculated. Finally, the following formula is used for calculation of the efficiency of the system:

$$\text{Efficiency} (\%) = \frac{P_2}{P_1} = \frac{\tau_2 \omega_2}{\tau_1 \omega_1} \cdot 100\%$$

![Figure 17: Chain losses suggested test set-up](image)

In the figure above one can see that electrical motor, two torque sensors and two rotation rate/speed sensors are needed for performing this test. It has to be noted that the test up used by the engine team to get a pressure trace uses an electrical motor and therefore, the test set up explained above can be realized. However, due to the schedule, it will not be possible to
perform the test before the project deadline but this can still be performed before the next competition.

**Static Test**

The second test that can be performed for measuring chain efficiency is a so called static test. It allows calculating the static efficiency as a ratio of the work out to the work in, where work is defined as the product of a force and the displacement it causes.

Test setup for this test can be seen in the figure on the right and it is less complex than the first test presented. The idea is to put a loop of chain over the chain wheel and to use steel hooks to hang a weight on each side of the chain so that they are balanced. By constantly adding small weights on the one side there will be a moment when the friction force will be overcome and the chain will start to rotate. By doing this the friction force can be measured. The efficiency in this case can be calculated using following formula:

\[
Efficiency(\%) = \frac{MgS}{(M + m_{added})gS} \cdot 100\% = \frac{M}{M + m_{added}} \cdot 100\%
\]

where:\n\n- \( M \) is a hanged weight and \( m_{added} \) is a small weight added.

The friction force may be not constant, due to engagement of the chain with the teeth, and probably some kind of force balance will be needed. By changing hanged masses different chain tensions can be achieved.

The important drawback of this technique is that the results obtained will not be accurate enough. That is the reason why the test has not been performed yet. However, this test could be performed in case of the need for a value even roughly correct to build the Matlab code would occur.

**b) Model Research**

So far, since nothing can be used as an input in the vehicle dynamic model regarding the chain losses, some research have been done to find a way to evaluate those later. However, the chain losses depends on so many parameters (chain tension, lubrication, positioning, engine speed...) that it would not be possible to choose one proper model and apply it to this specific case. This is due to the fact that all of these parameters are not known for Vera’s drive chain. A mathematical model has been found on internet. This model estimates the power needed along a complete drive line for a bicycle and takes into account all the losses. Hopefully, it should be possible to use it but the difficulty results in the fact that the model is too complex and the explanations are sometimes not so good or even missing.
8) Intermediate conclusion

As a half way conclusion before starting to build the vehicle dynamics model of Vera, one can say that the team succeeded in finding data for the tire rolling resistance, for bearing, one-way clutch and wheel rotational aerodynamic losses and those can be used as inputs into the model. The main data missing at this point are the losses coming from the chain. A test set-up has been previously suggested but it will not be performed within the automotive engineering project. Some models have been found to try to estimate a value for these losses but they are either too complicated or dependent on too many parameters so that it cannot be applied in Vera’s chain case. Therefore, an assumption on the chain efficiency will be done. Indeed, from the last year GPS data, the transmission efficiency was seen to be around 80%. That is the value that will be used in the following.

The next parts of the project are the implementation of a new vehicle dynamics model that will provide the best strategy for how the engine should be used during the race. Finally, some improvements will be suggested coming from the previous tests results presented in this report.
D. Vera’s dynamics model

This part of the report is aimed at explaining how the 2.5D vehicle dynamics model has been built in Matlab and how to use it. Furthermore, a short user’s instruction manual is included. The user has to know that the engine models used in the following stay unchanged compare to the previously implemented one. Only, the dynamics aspect of the model has been rebuilt from scratch. All the input used in this model are summarized in the table in appendix 8.

1) Instruction manual for the vehicle dynamics model

This following part can be seen as a user’s instruction manual for the Vera vehicle dynamics model. It will depict how the Matlab code should be used, what are the possibilities for the user and explain what the output results are. Furthermore, if there should be any evolution to bring to the model, the way to do it is included here as for instance for the banking angle of a track.

How to launch the simulation

After having opened Matlab software, the simulation can be launched by running the file “Vera_model.m”. Obviously, all the files have to be inside the workspace when running it. This m-file is the main function of this model and loads the different parameters (Vera, wheel, track profile...) as well as the strategy used for a simulation. Furthermore, it also calls the vehicle dynamics module in which equations of motion are included as well as the auxiliary function used to calculate the different loads and losses.

Track

In the model, two track profiles are available, Nogaro (France) and Lausitz (Germany). Both can be loaded easily under the comment “Loading of the track profile” by selecting 1 for Nogaro and 2 for Lausitz. The user has to know that the model takes into account the slope, the curve radius and the banking of the track. However, due to the lack of data regarding track profiles, some of them have not been taken into account by using assumptions.

For Nogaro, only the distance and altitude are available yet. Therefore, the curve radius is assumed to be infinite and the banking angle phi is set to 0. This is done automatically in the code when choosing Nogaro as the running track. Vera will then be considered heading on a perfectly straight line with a certain slope without any lateral load transfer.

For Lausitz, the curve radius as well as the slope is available for now but not the banking. The user should know that the model’s convention regarding the curve radius is that this latter is negative when Vera is turning left and positive when Vera is turning right. The banking angle is put to 0 here. If banking data are available later on, one should put them in the already defined vector phi below the comments “getting current track inclination” in the main function.

Strategies

Three different strategies have been implemented in the Matlab. The first one is a strategy optimized by playing around with speeds at which the engine has to be started and switched off. The second one is based on the same principle but with an add-on feature. Actually, it would be a waste of fuel to switch on the engine close to the finish line and end up the race
with an important unnecessary speed. However, it has to be noted that the average speed of Vera during the run should be greater than 30 km/h which is the minimum speed allowed by the Shell Eco-Marathon rules for a run to be validated.

**Number of laps**

The code allows the user to choose how many laps Vera should go through in one simulation. The variable to be changed is named “`lap_number`” in the reset variables part in the main function. When loading Lausitz track profile for instance, in order to have a full run simulation, this variable should be set to 7.

**Checking if the vehicle is going backwards**

This part has been kept from the previous model. This short function is aimed at stopping the simulation if the vehicle is going backwards. However, the previous condition regarding Vera’s speed was set to -5 m/s (which was very surprising!) to stop the simulation and say that Vera was going backwards. This condition has been obviously changed depending of the track. For Nogaro, the condition on Vera’s speed was set to -0.1. A safety margin has been taken to prevent the simulation from not starting. Actually, at race start, due to initial slope and assumed longitudinal wheel slip, Vera is slightly going backwards. For Lausitz, there is no problem at all so the Vera’s speed condition is set exactly to 0.

**Fuel lost when switching on the engine**

The code also takes into account the fact that there is some fuel lost at every engine start up. As there was no data available for the amount of this fuel an assumption was made. The variable for this can be found at `Load_Vera_parameters.m` and it is called `Vera.EngineStartFuelLoss`. The value assumed is $2.6 \times 10^{-6}$ liter at each engine engagement.

**Presentation of the results**

Once the simulation is over, the results are displayed to the user’s screen in two different ways. Some results are plotted in figures and some are displayed in Matlab command window. Five different figures are plotted and illustrate the following parameters for one simulation

**Figure 1:** Instantaneous speed of Vera versus the position in meter and mean speed (should be above 30 km/h).

**Figure 2:** Data relative to fuel consumption. The instantaneous fuel consumption is illustrated in green and the cumulative fuel consumption in blue. It has to be noted that the final value for the cumulative consumption is the total amount of fuel burnt during the run.

**Figure 3:** This figure shows the resistance moments occurring in function of the position on the track. This gives a first idea of the distribution of losses. The top graph illustrates the tire rolling resistance moment for each wheel. The mid-graph illustrates the frictional moment due to wheel bearings for each wheel and the bottom graph the one-way clutch moment for the rear wheel.

**Figure 4:** This figure depicts how the aerodynamic drag is split into body shell aerodynamic drag and rotational aerodynamic wheel drag. In the top graph, the aerodynamic drag force is illustrated for the shell with a drag coefficient of
0.108. In the bottom graph, the rotational aerodynamic drag for each wheel is plotted.

*Figure 5:* This figure is a diagram illustrating the final repartition of losses for the simulation. The first column on the left in blue is the energy produced by the engine. The others are representing the different losses. If one computes the summation of all the losses and the kinetic energy (if wheel slip is neglected) at which the vehicle crosses the finish line, the result should be the energy produced by the engine.

The remaining results can be found in the command window. The engine data, fuel data and speed data are available. The fuel data includes the total fuel consumption as well as the total distance that Vera can cover with 1L of gasoline. The speed data includes the total time, the mean speed and the final end speed of Vera crossing the finish line.

2) Model theoretical background

a) Preliminary details about the Vehicle Dynamics model:

It is a dynamic model which is capable of considering longitudinal and lateral effects. In the longitudinal direction (+ x-axis according to ISO standards), all transient effects are modeled (rotational inertia of the powertrain is excluded except the wheels); whereas in the lateral effects, a bicycle model for steady state cornering is used. The reason for using a steady state model for lateral dynamics is that Vera’s mass moment of inertia around z axis (ISO) is not known and it is not so practical to measure this concerning the resources available. The reason why a linear bicycle (single track) model used is that the steering angle for a given curve radius is wanted to be determined easily so as to make use of it in the steering resistance term (Otherwise, it will not be easy to extract steering angle from nonlinear equations. Since the steering angle is small, the accuracy of linear bicycle model is expected to be adequate). Even though a single track model is used, the load transfer is considered since tyre cornering stiffness does depend on vertical load and this dependence is not linear.

The equations of motion are written for four isolated bodies: Front left wheel, front right wheel, rear wheel and the chassis. They are given as in the following.

b) Rear Wheel

![Free body diagram of the rear wheel](image-url)
Equations of motion can be written as follows:

\[ \sum F_x = ma \]
\[ F_{T,r} - F_{Hr} - m_{\text{wheel}}g \sin \theta = m_{\text{wheel}}a_{\text{wheel}} = m_{\text{wheel}}a_x \]

\[ \sum F_z = ma \]
\[ W_r - N_{Hr} - mg \cos \theta = 0 \]

\[ \sum M = J \dot{\omega}_r \]
\[ T_{\text{wheel}} - M_{\text{res},r} - (F_{T,r} \times Rl) = J \dot{\omega}_r \]

where
\[ M_{\text{res},r} = M_{\text{roll},r} + M_{\text{bearing},r} + M_{\text{aero},r} + (M_{\text{OWC}}) \]

\[ M_{\text{OWC}} > 0 \text{ when } T_{\text{wheel}} = 0 \text{ (no engine torque applied)} \]
\[ M_{\text{OWC}} = 0 \text{ when } T_{\text{wheel}} > 0 \text{ (engine torque applied)} \]

Note that the translational acceleration of wheel is the same for translational acceleration of the vehicle chassis since elasticity of the chassis is neglected \( (a_{\text{wheel}} = a_x) \).

\[ \text{c) Front Left Wheel} \]

![Free Body Diagram of the Front Left Wheel](image)

Note that the free body diagram will remain the same for the front right wheel, only the subscripts will change from “fl” to “fr”.

Equations of motion can simply be written as follows:

\[ \sum F_x = ma \]
\[ F_{T,\beta} - F_{H,\beta} - m_{\text{wheel}}g \sin \theta = m_{\text{wheel}}a_{\text{wheel}} = m_{\text{wheel}}a_x \]

\[ \sum F_z = ma = 0 \]
\[ W_{f,\beta} - N_{H,\beta} - m_{\text{wheel}}g \sin \theta = 0 \]
\[ \Sigma M = J \dot{\omega}_{f,l} \]
\[ -M_{res,fl} - F_{T,fl} R_i = J \dot{\omega}_{f,l} \]

where
\[ M_{res,fl} = M_{roll,fl} + M_{bearing,fl} + M_{aero,fl} \]

Note that the translational acceleration of wheel is the same for translational acceleration of the vehicle chassis since elasticity of the chassis is neglected.

d) Front Right Wheel

As stated before, the free body diagram for this wheel is the same as the free body diagram of front left wheel except “fl” subscripts. Equations of motion can simply be written as follows:

\[ \Sigma F_x = ma \]
\[ F_{T,fr} - F_{H,fr} - m_{wheel} g \sin \theta = m_{wheel} a_{wheel} = m_{wheel} a_x \]
\[ \Sigma F_z = ma = 0 \]
\[ W_{fr} - N_{H,fr} - m_{wheel} g \sin \theta = 0 \]
\[ \Sigma M = J \dot{\omega}_{f,r} \]
\[ -M_{res,fr} - F_{T,fr} R_i = J \dot{\omega}_{f,r} \]

where
\[ M_{res,fr} = M_{roll,fr} + M_{bearing,fr} + M_{aero,fr} \]

Note that the translational acceleration of wheel is the same for translational acceleration of the vehicle chassis since elasticity of the chassis is neglected (a_{wheel} = a_x).

e) Body

![Figure 21: Free body diagram of the chassis](image)
To simplify the free body diagram, following conversions are performed:

\[ F_{Hf} = F_{H,fl} + F_{H,fr} \]
\[ N_{Hf} = N_{H,fl} + N_{H,fr} \]
\[ M_r = T_{\text{wheel}} - M_{\text{res,r}} \]
\[ M_f = (F_{\text{lateral,fo}} + F_{\text{lateral,fi}})R_i\sin\delta + M_{\text{res,fl}} + M_{\text{res,fr}} \]

Equations of motion for the chassis can be written as follows:

\[ \sum F_x = ma \]
\[ F_{H,fl} + F_{H,fr} + F_{Hr} - M_{\text{chassis}}g \sin \theta - F_{\text{aero}} - (F_{\text{lateral,fi}} + F_{\text{lateral,fo}})\sin \delta = M_{\text{chassis}}a_x \]

\[ \sum F_z = ma = 0 \]
\[ -M_{\text{chassis}}g \cos \theta + N_{H,fl} + N_{H,fr} + N_{Hr} = 0 \]

\[ \sum M_f = 0 \]
\[ -M_r + M_f - M_{\text{chassis}}g \sin \theta(h - R_i) - M_{\text{chassis}}a_x + N_{Hr}L - M_{\text{chassis}}g \cos \theta_f + (F_{\text{lateral,fi}} + F_{\text{lateral,fo}})R_i\sin \delta - F_{\text{aero}}(h_p - R_i) = 0 \]

f) Auxiliary expressions

To be able to proceed to the matrix model, auxiliary expressions should be written as follows:

- **Axle loads**

These equations play an important role in calculations of rolling resistances and bearing resistances:

\[ M_{\text{tot}} = M_{\text{chassis}} + 3m_{\text{wheel}} \]

\[ W_{fi} = M_{\text{tot}}g \frac{l_f}{2L} \cos \theta - M_{\text{tot}}g \frac{V^2}{R} - g \sin \phi \frac{l_f h}{L b} - M_{\text{tot}}g \frac{h}{2L} - M_{\text{tot}}g \sin \theta \frac{h}{2L} - F_{\text{aero}} \frac{h_p}{2L} \]

\[ W_{fo} = M_{\text{tot}}g \frac{l_f}{2L} \cos \theta + M_{\text{tot}}g \frac{V^2}{R} - g \sin \phi \frac{l_f h}{L b} - M_{\text{tot}}g \frac{h}{2L} - M_{\text{tot}}g \sin \theta \frac{h}{2L} - F_{\text{aero}} \frac{h_p}{2L} \]

\[ W_r = M_{\text{tot}}g \frac{l_r}{L} \cos \theta + M_{\text{tot}}g \frac{h}{L} + M_{\text{tot}}g \sin \theta \frac{h}{2L} + F_{\text{aero}} \frac{h_p}{L} \]

Note that since ISO coordinate system is used, front inner wheel turns out to be front left wheel when \( R > 0 \) (turning to the left). When \( R \) is negative (turning to the right), front inner wheel turns out to be front right wheel.
Another important issue here which is worth to mention here is the height of centre of pressure (h_p) used as a moment arm for the aerodynamic force. It is assumed to coincide with the stagnation point which is also assumed to be on the extremum point of the leading part (+x axis) of the shell. After some measurements, it was seen that h_p did approximate to 0.2 m.

- **Bearing resistance moment**

Two assumptions here are that the vehicle moves with a constant speed of 30 km/h and ambient conditions will be the same throughout the race conditions. This means that the speed does not influence bearing resistance moment too much. It has to be noted that there are two bearings for one wheel. Hence, an assumption had to be made regarding the axial forces carried by the outer and inner bearing. In this specific case, one can assume that the outer bearing takes the entire axial load and the inner one none of it. Therefore bearing resistance moments are in the following forms:

\[
M_{\text{bearing,ft}} = f \left( \sqrt{F_{H,ft}^2 + N_{H,ft}^2}, F_{\text{lateral,ft}} \right)
\]

\[
M_{\text{bearing,fr}} = f \left( \sqrt{F_{H,fr}^2 + N_{H,fr}^2}, F_{\text{lateral,fr}} \right)
\]

\[
M_{\text{bearing,rt}} = f \left( \sqrt{F_{H,rt}^2 + N_{H,rt}^2}, F_{\text{lateral,rt}} \right)
\]

- **Lateral forces**

Because the lateral acceleration level during the race is not so high, tyres can be assumed to retain their linear behaviour while cornering. As a result:

\[
F_{\text{lateral,fi}} = C_{\alpha,fi} \alpha_{fi}
\]

\[
F_{\text{lateral,fo}} = C_{\alpha,fo} \alpha_{fo}
\]

\[
F_{\text{lateral,ri}} = C_{\alpha,ri} \alpha_{ri}
\]

where

\[
C_{\alpha,fi} = (57.806 + 15.101P) \left[ 2\arctan \left( \frac{W_{fi}}{1000} \right) \right] \left( -0.082 + 0.186P \right)
\]

\[
C_{\alpha,fo} = (57.806 + 15.101P) \left[ 2\arctan \left( \frac{W_{fo}}{1000} \right) \right] \left( -0.082 + 0.186P \right)
\]
\[ C_{\alpha,r} = (57.806 + 15.101P) \left( 2 \arctan \left( \frac{W_r}{1000} \right) - 0.082 + 0.186P \right) \]

and assuming that for small steering angles, left and right steering angles are almost the same:

\[
\begin{align*}
\alpha_l &= \alpha_{fo} = \frac{180}{\pi} \left( \delta - \arctan \left( \frac{\Omega f}{V} \right) \right) \\
\alpha_r &= \frac{180}{\pi} \left( -\arctan \left( -\frac{\Omega r}{V} \right) \right) \\
&\approx \frac{180}{\pi} \left( \delta - \arctan \left( -\frac{l_r}{R} \right) \right)
\end{align*}
\]

Note that the cornering stiffness relations given above are taken from ETH book.

- **Steering angle**

From a simple steady state, linear bicycle model; steering angle can be easily calculated:

\[
\delta = \frac{L}{R} + \left( M_{tot} \frac{g l_f}{L} \frac{1}{C_{\alpha,fi} + C_{\alpha,fo}} (\pi/180) \right) - \left( M_{tot} \frac{g l_f}{L} \frac{1}{(\pi/180)} \right) \frac{V^2}{gR} \sin \phi
\]

Finally, aerodynamic resistance and rolling resistance could be written straight:

- **Aerodynamic resistance**

\[ F_{aero} = \frac{1}{2} \rho A C_d V^2 \]

- **Rolling resistance moment**

\[
\begin{align*}
M_{roll,fl} &= f_l W_{fl} R_l \\
M_{roll,fr} &= f_l W_{fr} R_l \\
M_{roll,rr} &= f_r W_r R_l
\end{align*}
\]

Here, it is assumed that rolling resistance coefficient does not change with vehicle speed and vertical load.
g) Matrix form

By combining equations presented, one can easily write them in matrix form. However, there are some important aspects in doing so. The static equations whose right hand side are equal to zero cannot be included in the matrix form, because they form linearly dependent system; as a consequence, the resulting matrix will be rank deficient (i.e. determinant is zero). Therefore, static equations in equations of motion are simply omitted in the matrix form. Instead, they are used as auxiliary equations to calculate road resistances.

Observing the equations of motion, one can count seven equations that are considered to be dynamic. However, there are ten unknowns to solve. This occurred because of lack of tyre data in longitudinal direction. In order to overcome this problem, longitudinal slip values for individual wheels are assumed. In “Theory of Ground Vehicles” book, the author says that for a passenger car, the wheel (driven) slip is around between 2% and 5% in ordinary driving conditions. Even though this value is given for passenger cars, it was thought that the interval could also be used for vehicle dynamics model of Vera. After a short brainstorming session, following longitudinal slip values were assumed:

When engine torque is applied:

\[ s_r = 3\% \]
\[ s_{\Omega} = s_{fr} = 0\% \]

When the vehicle is coasting down:

\[ s_r = s_{\Omega} = s_{fr} = 0\% \]

The reason why always zero slip assumed for the front wheels is that they have no driving torque, the only effect which creates slip on front wheels is the resistance torque on front wheels which can be considered as very small compared to drive torque.

By collecting all the dynamic equations from equations of motion for the wheels and the chassis, one could come up with the following:

\[
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & -1 & -m_{\text{wheel}} & \frac{J}{R_r(1-s_r)} & F_{T,\beta} \\
0 & 0 & R_j & 0 & 0 & 0 & \frac{F_{T,fr}}{R_r(1-s_r)} & \frac{F_{T,fr}}{J} & F_{T,fr} \\
1 & 0 & 0 & -1 & 0 & 0 & -m_{\text{wheel}} & \frac{J}{R_r(1-s_r)} & F_{H,\beta} \\
R_j & 0 & 0 & 0 & 0 & 0 & \frac{F_{H,fr}}{R_r(1-s_r)} & \frac{F_{H,fr}}{J} & F_{H,fr} \\
0 & 1 & 0 & 0 & -1 & 0 & -m_{\text{wheel}} & \frac{J}{R_r(1-s_r)} & F_{H,r} \\
0 & R_j & 0 & 0 & 0 & 0 & \frac{F_{H,r}}{R_r(1-s_r)} & \frac{F_{H,r}}{J} & a_x \\
0 & 0 & 0 & 1 & 1 & 1 & -M_{\text{chassis}} & & F_{\text{chassis}} \sin \theta + F_{\text{aero}} + (F_{\text{lateral,fr}} + F_{\text{lateral,fo}}) \sin \delta
\end{bmatrix}
= \begin{bmatrix}
m_{\text{wheel}}g \sin \theta \\
T_{\text{wheel}} - M_{\text{res,fr}} \\
m_{\text{wheel}}g \sin \theta \\
-M_{\text{res,fr}} \\
m_{\text{wheel}}g \sin \theta \\
-M_{\text{res,fr}} \\
M_{\text{chassis}}g \sin \theta + F_{\text{aero}} + (F_{\text{lateral,fr}} + F_{\text{lateral,fo}}) \sin \delta
\end{bmatrix}
\]
3) Map data processing for altitudes and cornering radii

Since some track data could not be found, some basic methods of scaling were to be used. The team was given a map of EuroSpeed Lausitz track map with detailed information about altitudes and also the GPS data from the last year racing. These two data sources were combined to get a more exact picture of how the vehicle performs on the track as it can be seen in the figure below.

![Figure 22: Map Scaling](image)

Having a map with exact altitudes and knowing that the track length is exactly 3200 meters (data obtained from Lausitz track webpage) the altitude profile of the track was obtained by measuring the map with a ruler and scaling this data. Altitude profile of the track is shown on the figure below. Using this profile the slope angle \( \theta \) for any lap position can be calculated.
For cornering radii calculation the form of the track was simplified to three straight lines and the three turns were assumed to be of constant curvature. Final curve radii can be found on the following figure.

4) Strategy selection

A number of different strategies were tested using the Vera dynamics model described previously. Strategy 0 tested was just about switching on the engine when the speed is below certain speed and switching it off when it exceeds a given limit. A modification of strategy 0 called strategy 1 has been implemented to the model. The principle is the same as strategy 0 except for the final part of the track close to the finish line, where the virtual driver is “forced”
not to accelerate to decrease the kinetic energy of the vehicle. Obviously, this should be done when the average speed if high enough in order it not to go down 30 km/h. Results for strategy 0 tests are summarized in the first table below. To date, as a part of regulation the average speed needs to be higher than 30 km/h, but due to aerodynamic forces that are proportional to the vehicle speed squared, it is reasonable to have a velocity as low as possible. So the team tried to have mean speed as low as it is allowed to be.

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As one can see the best result obtained with strategy 0 was 1339 km/L when having lower speed limit at 25 km/h and higher speed limit at 36 km/h. However, one should notice that the average speed is very close to 30 km/h and it would therefore be quite risky to choose this one. A good suggestion would be to switch on the engine when reaching a speed slightly greater than 25 and switch it off at a speed slightly greater than 36 km/h. Driving that way, the fuel consumption will increase a bit but the mean speed for a run will be higher than 30 km/h with a necessary safety margin.

The results for the strategy 1 with different ‘finish line’ parameters are given in the table below. The finish line parameter is the distance between his/her current position and the finish line within which the driver should not switch on the engine.

<table>
<thead>
<tr>
<th>Min speed, km/h</th>
<th>Max speed, km/h</th>
<th>Speed difference</th>
<th>Distance[m]/litre</th>
<th>Mean speed, km/h</th>
<th>Finish line,m</th>
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</thead>
<tbody>
<tr>
<td>26</td>
<td>36</td>
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<tr>
<td>25</td>
<td>37</td>
<td>12</td>
<td>1308</td>
<td>30.4</td>
<td>500</td>
</tr>
</tbody>
</table>

Actually the best result from the strategy 1 is the same as for strategy 0. This is because of the reason that strategies are very similar and for ‘finish line’ of 500 meters the engine was not working even in strategy 0. But for example for the case 26 - 36 km/h and finish line parameter of 500 meters the distance covered with one litre of gasoline was improved by more than 2% from 1300 m/L up to 1331 m/L when switching from strategy 0 to strategy 1. This latter is highly recommended even of the final performance is slightly lower since it is less risky due to the higher average speed.

Moreover, since the model includes steering resistance forces, another strategy has been tested. The strategy 2 is about accelerating after going through corners, so the vehicle pass
corners on the lowest speed. By doing so, losses for steering resistance are decreased but maximum speed of the vehicle increased up to 40 km/h. As aerodynamic drag dominates in general losses, the performance of the vehicle was not as good as it was achieved for strategy 1. The best result achieved was only 1220 km/L.

As a final recommendation it can be said that probably the best strategy will be to operate in the range with upper speed limit of 36 km/h and lower limit of 26 km/h. At the same time as the Vera is equipped with a bicycle computer the average speed should be monitored. When coming close to the finish line and knowing that value the decision about the need in acceleration should be made by the driver. This can help to pass the finish line with lower kinetic energy and thereby save some fuel. The speed curves for this case and losses distribution are presented on the figures below.

From this graph, it is obvious that the predominant resistance forces are the drag coming from the body shell and wheels drags as well as tire rolling resistance and chain losses. Therefore, in the following part, some design improvements will be suggested in order to reduce these losses. Steering resistance, bearing frictional losses and one-way clutch losses are of less importance and will not be dealt with in the following.
E. Suggested improvements

1) Wheel Rotational Aerodynamic Improvement

a) Introduction

The wheel rotational aerodynamic drag of the wheel is suspected to come from the spokes of the wheel, and the shear forces at the skin of the tire. To investigate the loss from spokes, the test setup of the rear wheel for one-way clutch and rotational aerodynamic was performed again with three different modifications by using bicycle speed sensor and logging system. Also, two methods of analysing (Law of Conservation of Energy, and Newton’s laws of motion) for One-way clutch and rotational aerodynamic have been used again in these tests.

b) Three modifications for the wheel

Three different modifications were suggested for improving the aerodynamic characteristics of the wheel. They can be seen on the following figures.

- First Modification (Wheel fully covered with plastic film)

![Figure 27: Modification 1 – Wheel fully covered with plastic film](image)

It was found that the first modification changes the shape of the wheel a lot and it was almost impossible to mount modified wheel on Vera because there were some stationary parts touching the plastic film.
- Second Modification (Spokes covered with plastic film)

The plastic film from the first modification has been cut to cover only spokes. By doing this the wheel can be mouthed in the rear axle and tested. Also the mass of the plastic cover has been measured, about 36 gram, is significantly lower than the mass of the wheel (~1300 grams) so the inertia added by these cover can be considered to be negligible. That means that all the changes in the frictional moment values correspond to the improvement in aerodynamics of the wheel. The three tests were performed for this modification.

- Third Modification (Spokes covered with paper sheet)

After testing the wheel with plastic film, it can be seen that the surface of it is very rough. To improve the surface the paper sheet has been used to cover the spokes. The three tests were performed for this modification.
c) Test results and post-processing

The graph shows the averaged results for three modifications.

![Graph showing results](image)

From the graph, it can be seen that covering the wheel with two different modifications can significantly improve the time for running from 40 to 20 km/h.

**From method 1 (Law of Conservation of Energy)**

From this method the aerodynamic drag coefficient can be obtained by using:

\[
\frac{\frac{1}{2}I_1 \omega_1^2 - \frac{1}{2}I_2 \omega_2^2 - \int_{t_1}^{t_2} B \omega dt}{\int_{t_1}^{t_2} \omega^2 dt}; \text{ by taken bearing moment losses as } 0.000265 \text{ N.m (average of bearing moment losses from previous test)}, \text{ OWC } = 0.00361 \text{ N.m (average from previous test)}
\]

The bar chart below shows the aerodynamic constant of the wheel for original wheel and the wheel with two modifications.

![Bar chart showing aerodynamic constants](image)
From the graph, the aerodynamic drag constant for original wheel is slightly different from the result from one-way clutch and wheel aerodynamic test in “Collecting Vera data” chapter. This is because the axle load of the bearing changes due to tension force of the axle when assembly it effects the aerodynamic drag constant calculation. For the improvement, it can be seen that the aerodynamic drag constant for paper covered decreases almost twice compared to the original wheel.

**From method2 (Newton’s laws of motion)**

From this method, the losses moment due to aerodynamic resistance can be calculated by plotting $I_1 \omega$ versus $\omega$.

The graph shows the values of the frictional moment versus rotational speed.

![Graph showing frictional moment versus rotational velocity](image)

Figure 32: Frictional moment versus rotational velocity

As one can see general frictional moment at the velocity of 40 km/h was decreased from 3.33e-2 Nm to 1.920e-2 Nm in other words by 42% and for the velocity of 20km/h the moment was decreased from 1.248e-2 Nm to 8.28e-3 Nm or by 34%. This is a significant decrease in general frictional moment, but it should be said that, if considering all other losses not changing from experiment to experiment, the decrease in aerodynamic losses is even higher. That means that the improvement work pretty well.

**The distribution of frictional moment losses**

As a final result, having one way clutch losses calculated and bearing losses distribution the final frictional moment distribution due to different losses can be plotted for the best case (paper covered), it can be found on the next figure.
As shown in the graph above, after wheel being modified using paper cover, the aerodynamic of the wheel still plays the major role in losses at the rear axle.

**Improvement from Vera vehicle dynamic model (Matlab model)**

The original wheel and the wheel covered with paper are simulated in the vehicle dynamic model to investigate the benefit of covering the wheels. The running strategy is to start up the engine when the vehicle speed is below 25 km/h and shut down when it over than 38 km/h. The energy losses and the distance per 1 liter of gasoline in both wheels will be investigated. It can be seen from the figure below that the wheel aerodynamic drag energy is reduced by 61.47%
Also, the distance after simulated in the model for original wheel is about 1300 km/L and for the wheel covered with paper is about 1353 km/L. The improvement of the paper covered is that the vehicle can go 53 km more in the simulation.

2) Vera’s body shell improvement suggestion

Even though Vera’s body should remain untouched for this year’s competition, the drag resistance force is dominant over an entire run as seen from the simulations. Therefore, a suggestion in order to decrease this drag loss had to be made and will be presented here.

One knows that there are two main different types of drag which are called pressure drag and friction drag. The friction drag is predominant in streamlined vehicles and is due to the friction between the air and the body shell. Vera is a streamlined vehicle and therefore, the friction drag is the dominant one. In order to decrease it a bit, the ground clearance could be increased as explained in the “The World’s Most Fuel Efficient Vehicle” book.

Even if the pressure drag is not the predominant one in Vera’s case, a pressure difference between the front and the rear end does still exist due to the separation between the shell and the flow as this latter approaches the rear end. Therefore, an idea would be to add vortex generators on Vera’s shell. These vortex generators are devices that have been effectively used in the aeronautical industry to improve lift and reduce drag of aerodynamic surfaces by delaying the flow separation. The principle behind this is that these devices produce strong vortices that transfer momentum from the outer most regions to the lower regions of the boundary layer. Therefore, the shell attached flow is maintained at positive pressure gradient, preventing the flow from separating for a certain time.

In the automotive industry, there are several vortex generator designs that have already been tested in the literature and also available in the market, from simple thin triangular shaped surfaces to more elaborate designs. It has to be noted that some studies performed on passenger cars claim that those designs, if well-adjusted, lead to a narrower wake downstream and hence less pressure drag. However, some studies claim that it works in theory only but after testing, the addition of vortex generator was of insignificant effect on the air flow.

In spite of the fact that this idea is not fully validated, a suggestion would be to add some vortex generators at the rear end of Vera as illustrated below in order to delay the flow separation and increase the pressure at the rear end. The vortex generators could be designed in the typical form of NACA type inlet profiles with a height comparable to the boundary layer thickness in their area of location. An example of vortex generators for passenger car (NACA inlet type) is illustrated in the left picture below.

![Figure 34: Vortex generators example](image-url)
3) Toothed belt

Unfortunately, because of lack of time and resources, chain drive installed on Vera could not be tested. Previously, it was known that the drivetrain efficiency was around 80% (This was stated in the code). This is not acceptable for a competition which aims to reveal how far a vehicle can go with a given amount of energy. The biggest loss is believed to be caused by chain tensioner. After a short investigation on the mechanism, it was found that the bearing which supports the tensioning sprocket could not be rotated easily. It needs therefore to be replaced, but this should be done properly. First of all, what has gone wrong to damage the bearings should be identified. It is known that efficient bearings may no longer perform satisfactorily if tight fitting is used to fit the bearings. However, loose fitting may simply lead the tensioning sprocket to come off! As a result, bearing should be supported axially from its two sides. One solution may be to use Seeger rings. If, after investigations, it is found that the bearing is damaged because of excessive bending moment, then simply number of bearings holding the tensioning sprocket should be increased axially.

Concerning the efficiency of the transmission system, it is “guessed” that a properly designed belt system is more efficient in overall compared to chain systems. One can think that this statement holds when toothed belt and pulley systems are employed, since form closed (form conditioned) transmission systems are more efficient than friction closed (friction conditioned) transmission systems. According to “Makine Elemanları ve Konstrüksiyon Örnekleri” book and Continental’s extensive technical document about “CONTI SYNCHROBELT HTD Synchronous Drive Belts”, it is stated that these belt systems have efficiencies up to 98%. According to ETH book, the efficiency of Powergrip HTP synchronous belt systems goes up to 99%. Also in ETH book, it is stated that their solution to transmit power from the engine to the wheel in PAC-Car I (the first version of their record breaking vehicle) was to use a belt drive (In record breaking vehicle, PAC-Car II, gear drive was used, still not the chain!).

Under these circumstances, it is natural to think that a properly designed belt drive will be more efficient than a chain drive. The advantages of the belt systems according to ETH book are:

- Lightweight design
- Low cost
- Low weight
- No need for lubrication

Moreover, unlike chain systems, the belt drives do not suffer from “polygon effect” which is simply the rotational speed variation due to nonflexible nature of each chain link. This phenomenon is suspected to increase the vibrations and consequently the energy loss.

As expected, tensioning is required also for the belt systems even though the amount of tensioning required is less in the toothed belt drives compared to the other belt drives. This can be achieved by using the current tensioning system (but after the problem with its bearing is solved!) but replacing the freewheeling sprocket with a freewheeling pulley. If the belt is not desired to remain tensioned all the time, a special construction, in which the required tensioning force is obtained when engine delivers the power, can be employed. The schematic figure is shown in the following:
By utilising the special construction shown above, one can prevent the belt from being tensioned all the time (This will affect the lifetime and bearing losses).

After all, if the future team is convinced enough to select a belt drive, they could refer to Continental’s extensive technical document about “CONTI SYNCHROBELT HTD Synchronous Drive Belts” which includes detailed information about calculations and selection of the properly dimensioned belt drive systems (including pulley).

4) Tubeless tires

As can be seen from the MATLAB simulation, rolling resistance due to the tyres constitutes one of the biggest energy losses. This means that there is quite a lot of space here to improve the rolling resistance of the tyres. However, because the “Eco tyres” are provided by tyre manufacturer, the teams cannot change the structure of them. Indeed, recent “Eco tyres” are well optimised for rolling resistance. However, after testing tyres for rolling resistance, it was seen that the rolling resistance coefficient did not correspond to the rolling resistance value given in the ETH book. This may arise from several possibilities:

- The presence of confounding factors (such as presence of traction force when testing rolling resistance) inherent in the test setup
- Damaged tyres (Tyres remained warped even after inflating them with 5 bars of pressure)
- The tube inside the tyre (Because the rolling resistance for this tyre given in the literature is valid for tubeless construction)
It is known from the literature and also intuitively that presence of tube increases the rolling resistance because of relative motion between tyre and tube while the thread is in contact with ground. In order to be able to remove the tube from the construction, a new type of rim should be designed. If future team has both enough time and eagerness to construct this type of rim, ETH book (pp. 161-162) provides all the technical details (taken from “European Tyre and Rim Technical Dimensions”) about the suitable rim contour which allows current tyres to be used without a tube.

5) Wheelhouses taping

The final suggestion that should be done is basically to put some scotch tape to fill wheelhouses’ gaps between wheels and body shell. This will prevent the airflow from coming inside the shell and therefore reduce the aerodynamic drag of a considerable amount. As stated in the document “Implementing advanced CAE tools in automotive engineering education at Chalmers University” where a shell CFD simulation has been performed, the main shell locations for a drag improvement are wheelhouses. Therefore, taping the wheelhouses would be a good way to decrease the drag. Since no CFD simulation have been performed with no gap but for wheels, the corresponding new drag coefficient is not known and it was not possible to verify how much the fuel consumption could be decreased using the vehicle dynamics model.
Conclusion

To conclude, the team succeeded in finding data for the rolling resistance, for bearing losses, for one-way clutch losses and rotational wheel aerodynamic drag losses. The data missing are the losses coming from the chain due to the complicated test setup which required the dynamo and rotational speed meters as described previously in the suggested test. Also, the team has built a Vera vehicle dynamics model (2.5D) including all the previously stated losses. The model takes into account longitudinal and lateral load transfers. By using this model, its results are close to the results from the real competition and all energy losses distribution diagram has been plotted. It shows that the main losses are from longitudinal shell aerodynamic drag, tire rolling resistance and chain drive system. Moreover, the driving strategies are very important in order to improve the fuel consumption and increase the distance Vera can cover with one liter of gasoline. To start and stop the engine when needed and keep the maximum speed as low as possible seem to be the best strategies. The best strategy has been defined and optimized and should be used for the coming competition. Also, covering the wheel by using paper sheets can be one of the suggested design improvements. This technique can help to reduce the rotational aerodynamic of the wheels as seen previously. Minimizing the chain losses by using belt system or at least investigating the condition of bearing of chain tensioner might increase the efficiency of drive system. Moreover, modifying the body shell of Vera can be done to reduce the aerodynamic drag since this is the main energy loss, and the rolling resistance might be reduced by using tubeless tire. Finally, the team has reached the goal statement by optimizing Vera from Vehicle Dynamic point of view, analyzing the different losses, coming with design improvement, and working as a team.
## Appendices

1) **APPENDIX 1: Planning of the project**

<table>
<thead>
<tr>
<th>Task</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</table>

**Legend:**
- ■ Overall planning
- ■ Detailed planning

*Figure 36: Final planning*
## 2) APPENDIX 2: Risk analysis

<table>
<thead>
<tr>
<th>N</th>
<th>Event</th>
<th>Severity 1-10</th>
<th>Probability 1-10</th>
<th>Risk number</th>
<th>Actions to reduce the risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Driver is not available for the test</td>
<td>1</td>
<td>6</td>
<td>6</td>
<td>Mai/Caroline can drive/sit in the car.</td>
</tr>
</tbody>
</table>
| 2  | Bad weather/Not suitable road surface for testing                     | 7             | 9                | 63          | 1. For bad weather, rolling test should be scheduled as early as possible.  
                                  |               |                  |              | 2. For surface condition, use the parking lot here. |
| 3  | E-mail delivery problems/delays                                       | 2             | 3                | 6           | If this is suspected, contact team members via mobile phone. |
| 4  | Absence of team member(s)                                             | 3             | 9                | 27          | 1. Absent member(s) can be informed about what is achieved/decided that day through the meeting minutes.  
                                  |               |                  |              | 2. If a team member is absent that day and he/she is supposed to prepare the test setup, then absent member can send the experiment planning to the group so that other members can conduct the experiment. |
| 5  | Data loss                                                             | 9             | 3                | 27          | 1. Distribute the data among other team members  
                                  |               |                  |              | 2. Create backups |
| 6  | Misunderstanding/Disagreement/Argument between group members           | 3             | 10               | 30          | 1. To avoid misunderstandings, detailed meeting minutes could be written. Additional discussions can also help the members to understand everything properly.  
                                  |               |                  |              | 2. In case of a conflict/argument/disagreement, project leader takes over and chooses what to do. |
| 7  | Someone fails to deliver his/her responsibility or does not respect deadline | 5             | 10               | 50          | According to game rules; if this happens, then he/she will have to order a cup of coffee or equivalent for each group member. |
| 8  | Vera is not available at the time of an experiment                    | 3             | 5                | 15          | 1. Immediately try to schedule the test for another day.  
                                  |               |                  |              | 2. Deal with the next task immediately in order not to lose time.  
                                  |               |                  |              | 3. Inform the engine team about the schedule of experiments so their experiments will not coincide with ours. |
| 9  | Final validation tests are needed to be scheduled in the winter, bad weather problems! | 6             | 9                | 54          | Try to find an equivalent closed test track. |
3) APPENDIX 3: Rolling resistance test results

Results of rolling resistance on wet asphalt from 5°, 8° or 10° of start angle until the wheel stops:

<table>
<thead>
<tr>
<th>Wet</th>
<th>Angle</th>
<th>5°</th>
<th>8°</th>
<th>10°</th>
<th>5°</th>
<th>8°</th>
<th>10°</th>
<th>5°</th>
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<td>1</td>
<td>2</td>
<td>3</td>
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<td>38</td>
<td>37</td>
<td>37.33</td>
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<tr>
<td></td>
<td>Attenuation Time</td>
<td>28.2</td>
<td>24.9</td>
<td>25.5</td>
<td>24.4</td>
<td>25.75</td>
<td>28.2</td>
<td>29.5</td>
<td>28.9</td>
<td>28.87</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Avg.</td>
<td>34.25</td>
<td>25.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>28.2</td>
<td>29.5</td>
<td>28.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>32.1</td>
<td>31.9</td>
<td>30.7</td>
<td>32.1</td>
<td>31.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Results of rolling resistance on dry asphalt from a start angle of 20° to 10°:

<table>
<thead>
<tr>
<th>Asphalt</th>
<th>Angle</th>
<th>10° to 20°</th>
<th>20° to 10°</th>
<th>Aver.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test number</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Period Number</td>
<td>10</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Attenuation Time</td>
<td>8.45</td>
<td>7.76</td>
<td>7.68</td>
<td>7.48</td>
</tr>
<tr>
<td>Rolling Resistance Coefficient</td>
<td>0.00226</td>
<td>0.00251</td>
<td>0.00251</td>
<td>0.00251</td>
</tr>
</tbody>
</table>

Results of rolling resistance on dry asphalt from a start angle of 20° to 10° but with the lower bar removed (so the centre of gravity is higher):

<table>
<thead>
<tr>
<th>Test number</th>
<th>Elapsed Time (s)</th>
<th>Number of periods</th>
<th>Rolling resistance coeff</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>13.04</td>
<td>10</td>
<td>0.001535</td>
</tr>
<tr>
<td>T2</td>
<td>11.48</td>
<td>9</td>
<td>0.001705</td>
</tr>
<tr>
<td>T3</td>
<td>12.93</td>
<td>10</td>
<td>0.001535</td>
</tr>
<tr>
<td>T4</td>
<td>11.56</td>
<td>9</td>
<td>0.001705</td>
</tr>
<tr>
<td>T5</td>
<td>11.59</td>
<td>9</td>
<td>0.001705</td>
</tr>
<tr>
<td>T6</td>
<td>12.92</td>
<td>10</td>
<td>0.001535</td>
</tr>
<tr>
<td>T7</td>
<td>13.03</td>
<td>10</td>
<td>0.001535</td>
</tr>
<tr>
<td>Average</td>
<td>12.36428571</td>
<td>9.571428571</td>
<td>0.001608</td>
</tr>
</tbody>
</table>
4) APPENDIX 4: Simulink model of the tire rolling resistance

Figure 37: Tire Resistance Model overview
### 5) APPENDIX 5: Wheel inertia measurement results

#### M = 45 g

<table>
<thead>
<tr>
<th>S = 1 meter</th>
<th>Time (s)</th>
<th>Linear Acceleration (m/s²)</th>
<th>Inertia (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position 1 (0 degree)</td>
<td>1.81</td>
<td>0.6105</td>
<td>0.03825</td>
</tr>
<tr>
<td></td>
<td>1.96</td>
<td>0.5206</td>
<td>0.04529</td>
</tr>
<tr>
<td></td>
<td>1.73</td>
<td>0.6682</td>
<td>0.03472</td>
</tr>
<tr>
<td>Position 2 (90 degrees)</td>
<td>2.00</td>
<td>0.5000</td>
<td>0.04726</td>
</tr>
<tr>
<td></td>
<td>1.98</td>
<td>0.5102</td>
<td>0.04627</td>
</tr>
<tr>
<td></td>
<td>1.96</td>
<td>0.5206</td>
<td>0.04529</td>
</tr>
<tr>
<td>Position 3 (180 degrees)</td>
<td>1.68</td>
<td>0.7086</td>
<td>0.03260</td>
</tr>
<tr>
<td></td>
<td>1.84</td>
<td>0.5907</td>
<td>0.03961</td>
</tr>
<tr>
<td></td>
<td>1.84</td>
<td>0.5907</td>
<td>0.03961</td>
</tr>
<tr>
<td>Position 4 (270 degrees)</td>
<td>1.82</td>
<td>0.6038</td>
<td>0.03870</td>
</tr>
<tr>
<td></td>
<td>1.89</td>
<td>0.3599</td>
<td>0.04194</td>
</tr>
<tr>
<td></td>
<td>1.76</td>
<td>0.6457</td>
<td>0.03603</td>
</tr>
</tbody>
</table>

**Average Inertia**: 0.04047  
**Standard deviation**: 0.00479  
**Confidence Interval (95%)**: 0.00271  
**Inertia Max**: 0.04318  
**Inertia Min**: 0.03775

#### M = 75 g

<table>
<thead>
<tr>
<th>S = 1.1 meter</th>
<th>Time (s)</th>
<th>Linear Acceleration (m/s²)</th>
<th>Inertia (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position 1 (0 degree)</td>
<td>1.43</td>
<td>1.0758</td>
<td>0.03434</td>
</tr>
<tr>
<td></td>
<td>1.48</td>
<td>1.0044</td>
<td>0.03709</td>
</tr>
<tr>
<td></td>
<td>1.46</td>
<td>1.0321</td>
<td>0.03598</td>
</tr>
<tr>
<td>Position 2 (90 degrees)</td>
<td>1.51</td>
<td>0.9649</td>
<td>0.03878</td>
</tr>
<tr>
<td></td>
<td>1.59</td>
<td>0.8702</td>
<td>0.04346</td>
</tr>
<tr>
<td></td>
<td>1.56</td>
<td>0.9040</td>
<td>0.04168</td>
</tr>
<tr>
<td>Position 3 (180 degrees)</td>
<td>1.6</td>
<td>0.8594</td>
<td>0.04406</td>
</tr>
<tr>
<td></td>
<td>1.54</td>
<td>0.9276</td>
<td>0.04051</td>
</tr>
<tr>
<td></td>
<td>1.59</td>
<td>0.8702</td>
<td>0.04346</td>
</tr>
<tr>
<td>Position 4 (270 degrees)</td>
<td>1.53</td>
<td>0.9398</td>
<td>0.03993</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>0.9778</td>
<td>0.03821</td>
</tr>
<tr>
<td></td>
<td>1.56</td>
<td>0.9040</td>
<td>0.04168</td>
</tr>
</tbody>
</table>

**Average Inertia**: 0.03993  
**Standard deviation**: 0.00313  
**Confidence Interval (95%)**: 0.00177  
**Inertia Max**: 0.04170  
**Inertia Min**: 0.03816
<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Linear Acceleration ( (\text{m/s}^2) )</th>
<th>Inertia ( (\text{kg.m}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position 1 (0 degree)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.29</td>
<td>1.3220</td>
<td>0.03621</td>
</tr>
<tr>
<td>1.32</td>
<td>1.2626</td>
<td>0.03818</td>
</tr>
<tr>
<td>1.37</td>
<td>1.1721</td>
<td>0.04157</td>
</tr>
<tr>
<td>Position 2 (90 degrees)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.39</td>
<td>1.1387</td>
<td>0.04296</td>
</tr>
<tr>
<td>1.35</td>
<td>1.2071</td>
<td>0.04020</td>
</tr>
<tr>
<td>1.34</td>
<td>1.2252</td>
<td>0.03952</td>
</tr>
<tr>
<td>Position 3 (180 degrees)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.29</td>
<td>1.3220</td>
<td>0.04296</td>
</tr>
<tr>
<td>1.39</td>
<td>1.1387</td>
<td>0.04296</td>
</tr>
<tr>
<td>1.37</td>
<td>1.1721</td>
<td>0.04157</td>
</tr>
<tr>
<td>Position 4 (270 degrees)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.39</td>
<td>1.1387</td>
<td>0.04296</td>
</tr>
<tr>
<td>1.40</td>
<td>1.1224</td>
<td>0.04366</td>
</tr>
<tr>
<td>1.42</td>
<td>1.0911</td>
<td>0.04508</td>
</tr>
</tbody>
</table>

Average Inertia 0.04092
Standard deviation 0.00290
Confidence Interval (95%) 0.00164
Inertia Max 0.04256
Inertia Min 0.03928

---

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Linear Acceleration ( (\text{m/s}^2) )</th>
<th>Inertia ( (\text{kg.m}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position 1 (0 degree)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.26</td>
<td>1.3857</td>
<td>0.04458</td>
</tr>
<tr>
<td>1.26</td>
<td>1.3857</td>
<td>0.04458</td>
</tr>
<tr>
<td>1.21</td>
<td>1.5026</td>
<td>0.04054</td>
</tr>
<tr>
<td>Position 2 (90 degrees)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.21</td>
<td>1.5026</td>
<td>0.04054</td>
</tr>
<tr>
<td>1.17</td>
<td>1.6071</td>
<td>0.03743</td>
</tr>
<tr>
<td>1.17</td>
<td>1.6071</td>
<td>0.03743</td>
</tr>
<tr>
<td>Position 3 (180 degrees)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.14</td>
<td>1.6928</td>
<td>0.03516</td>
</tr>
<tr>
<td>1.21</td>
<td>1.5026</td>
<td>0.04054</td>
</tr>
<tr>
<td>1.21</td>
<td>1.5026</td>
<td>0.04054</td>
</tr>
<tr>
<td>Position 4 (270 degrees)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.17</td>
<td>1.6071</td>
<td>0.03743</td>
</tr>
<tr>
<td>1.23</td>
<td>1.4542</td>
<td>0.04214</td>
</tr>
<tr>
<td>1.21</td>
<td>1.5026</td>
<td>0.04054</td>
</tr>
</tbody>
</table>

Average Inertia 0.04012
Standard deviation 0.00288
Confidence Interval (95%) 0.00163
Inertia Max 0.04175
Inertia Min 0.03849
6) **APPENDIX 6: Pendulum parts drawings**

Figure 38: Pendulum steel plate drawing

Figure 39: Pendulum steel bar drawing
### 7) APPENDIX 7: Vera data table

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Type</th>
<th>Additional Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weight of vehicle</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total ladden weight</td>
<td>80.6</td>
<td>kg</td>
<td>Measurement</td>
<td>Measurement with Shasha Xie inside. Simple scales have been used</td>
</tr>
<tr>
<td>Ladden weight w/o bodywork</td>
<td>74.5</td>
<td>kg</td>
<td>Measurement</td>
<td>Measurement with simple scales</td>
</tr>
<tr>
<td>Bodywork weight</td>
<td>6.1</td>
<td>kg</td>
<td>Measurement</td>
<td>Measurement with simple scales</td>
</tr>
<tr>
<td>Front Bodywork weight</td>
<td>3.8</td>
<td>kg</td>
<td>Measurement</td>
<td>Measurement with simple scales</td>
</tr>
<tr>
<td>Rear Bodywork weight</td>
<td>2.3</td>
<td>kg</td>
<td>Measurement</td>
<td>Measurement with simple scales</td>
</tr>
<tr>
<td>Weight front left wheel</td>
<td>24.3</td>
<td>kg</td>
<td>Measurement</td>
<td>Measurement with Shasha Xie inside. Simple scales have been used</td>
</tr>
<tr>
<td>Weight front right wheel</td>
<td>24.8</td>
<td>kg</td>
<td>Measurement</td>
<td>Measurement with Shasha Xie inside. Simple scales have been used</td>
</tr>
<tr>
<td>Weight rear wheel</td>
<td>25.4</td>
<td>kg</td>
<td>Measurement</td>
<td>Measurement with Shasha Xie inside. Simple scales have been used</td>
</tr>
<tr>
<td><strong>Vehicle Dimensions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wheelbase</td>
<td>1410</td>
<td>mm</td>
<td>Measurement</td>
<td>Measured with a ruler</td>
</tr>
<tr>
<td>Centre of gravity length if</td>
<td>481</td>
<td>mm</td>
<td>Calculation</td>
<td>Distance between the front axle and the cog</td>
</tr>
<tr>
<td>Centre of gravity length lr</td>
<td>929</td>
<td>mm</td>
<td>Calculation</td>
<td>Distance between the rear axle and the centre of gravity</td>
</tr>
<tr>
<td>Centre of gravity Height</td>
<td>223</td>
<td>mm</td>
<td>Calculation</td>
<td>Calculation made from test with raised front axle (laden vehicle)</td>
</tr>
<tr>
<td>Trackwidth</td>
<td>510</td>
<td>mm</td>
<td>Measurement</td>
<td>Measured with a ruler</td>
</tr>
<tr>
<td><strong>Wheel Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unloaded wheel radius</td>
<td>237.5</td>
<td>mm</td>
<td>Measurement</td>
<td>Measured with a ruler and calculated from the tire dimensions</td>
</tr>
<tr>
<td>Loaded wheel radius</td>
<td>236</td>
<td>mm</td>
<td>Measurement</td>
<td>With 1.5 mm of deflection assumed</td>
</tr>
<tr>
<td>Wheel weight</td>
<td>1.175</td>
<td>kg</td>
<td>Measurement</td>
<td>Measured with the scale from the workshop (should be accurate enough)</td>
</tr>
<tr>
<td>Wheel thickness</td>
<td>40</td>
<td>mm</td>
<td>Measurement</td>
<td>Measured with a ruler</td>
</tr>
<tr>
<td>Tire inflation pressure</td>
<td>5</td>
<td>bar</td>
<td>Measurement</td>
<td>Pressure of the tires during the competition</td>
</tr>
<tr>
<td>Rolling Resistance of the</td>
<td>0.0024</td>
<td>-</td>
<td>Measurement</td>
<td>Measured with the pendulum test</td>
</tr>
<tr>
<td>tires</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inertia of the wheel</td>
<td>0.0404</td>
<td>kg.m²</td>
<td>Measurement</td>
<td>Measured with the inertia test</td>
</tr>
<tr>
<td><strong>Aerodynamic Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frontal area</td>
<td>0.32</td>
<td>m²</td>
<td>Estimation</td>
<td>Estimated</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>0.108</td>
<td>-</td>
<td>Measurement</td>
<td>Simulation done by the aerodynamic department (without wheels)</td>
</tr>
<tr>
<td><strong>Bearing Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient wheel bearing</td>
<td>0.007</td>
<td>-</td>
<td>Calculation</td>
<td>0.001-0.002 in excellent conditions, with 0.007 the results is equal to SKF simulations</td>
</tr>
<tr>
<td>Bearing bore diameter</td>
<td>0.015</td>
<td>m</td>
<td>Measurement</td>
<td>The smaller the better</td>
</tr>
<tr>
<td><strong>Various Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transmission Efficiency</td>
<td>0.8</td>
<td>-</td>
<td>Estimation</td>
<td>From GPS data</td>
</tr>
<tr>
<td>Clutch engine speed</td>
<td>1700</td>
<td>RPM</td>
<td>Estimation</td>
<td>RPM where the clutch engages</td>
</tr>
<tr>
<td>Clutch time start up</td>
<td>0.3</td>
<td>s</td>
<td>Estimation</td>
<td>Time before the clutch engages</td>
</tr>
<tr>
<td><strong>Pendulum Device</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dimensions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total pendulum weight</td>
<td>39.45</td>
<td>kg</td>
<td>Measurement</td>
<td>Measured with the scale from the workshop</td>
</tr>
<tr>
<td>Main rods weight</td>
<td>15.9</td>
<td>kg</td>
<td>Measurement</td>
<td>Measured with the scale from the workshop</td>
</tr>
<tr>
<td>Main rods length</td>
<td>400</td>
<td>mm</td>
<td>Measurement</td>
<td>Manufactured with this exact diameter</td>
</tr>
<tr>
<td>Main rods diameter</td>
<td>80</td>
<td>mm</td>
<td>Measurement</td>
<td>Manufactured with this exact diameter</td>
</tr>
<tr>
<td>Plates weight</td>
<td>2.65</td>
<td>kg</td>
<td>Measurement</td>
<td>Measured with the scale from the workshop</td>
</tr>
<tr>
<td>Plates thickness</td>
<td>15</td>
<td>mm</td>
<td>Measurement</td>
<td>Manufactured with this exact diameter</td>
</tr>
</tbody>
</table>
8) APPENDIX 8: Parameters used in the Vehicle dynamics model

<table>
<thead>
<tr>
<th><strong>Vera parameters</strong></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloaded Mass</td>
<td>30,6</td>
<td>kg</td>
</tr>
<tr>
<td>Pilot Mass</td>
<td>50</td>
<td>kg</td>
</tr>
<tr>
<td>Total Mass</td>
<td>80,6</td>
<td>kg</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>1,41</td>
<td>m</td>
</tr>
<tr>
<td>Track width</td>
<td>0,51</td>
<td>m</td>
</tr>
<tr>
<td>Center of Gravity height</td>
<td>0,223</td>
<td>m</td>
</tr>
<tr>
<td>Lf</td>
<td>0,481</td>
<td>m</td>
</tr>
<tr>
<td>Lr</td>
<td>0,929</td>
<td>m</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>150/11</td>
<td>[-]</td>
</tr>
<tr>
<td>Shell Drag coefficient</td>
<td>0,108</td>
<td>[-]</td>
</tr>
<tr>
<td>Pressure center height</td>
<td>0,2</td>
<td>m</td>
</tr>
<tr>
<td>Frontal Area</td>
<td>0,32</td>
<td>m²</td>
</tr>
<tr>
<td>Chain efficiency</td>
<td>80</td>
<td>%</td>
</tr>
<tr>
<td>Bearing Diameter</td>
<td>0,015</td>
<td>m</td>
</tr>
<tr>
<td>Slipping clutch engine speed</td>
<td>1700</td>
<td>RPM</td>
</tr>
<tr>
<td>Slipping clutch time</td>
<td>0,3</td>
<td>s</td>
</tr>
<tr>
<td>One-way clutch resisting moment</td>
<td>0,00361</td>
<td>N.m</td>
</tr>
<tr>
<td>Fuel lost when engine switched on</td>
<td>2,7*10^-6</td>
<td>L</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Wheel parameters</strong></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel radius</td>
<td>0,236</td>
<td>m</td>
</tr>
<tr>
<td>Inflation pressure</td>
<td>5</td>
<td>bars</td>
</tr>
<tr>
<td>Wheel mass</td>
<td>1,175</td>
<td>kg</td>
</tr>
<tr>
<td>Tire rolling resistance coefficient</td>
<td>0,0019</td>
<td>[-]</td>
</tr>
<tr>
<td>Wheel Inertia</td>
<td>0,0404</td>
<td>kg.m²</td>
</tr>
<tr>
<td>Longitudinal front slip</td>
<td>0</td>
<td>[-]</td>
</tr>
<tr>
<td>Longitudinal rear slip</td>
<td>0,03</td>
<td>[-]</td>
</tr>
<tr>
<td>Wheel aerodynamic coefficient</td>
<td>1,55*10^-5</td>
<td>[-]</td>
</tr>
</tbody>
</table>
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Input torque [Nm], tension force [N]</td>
</tr>
<tr>
<td>τ</td>
<td>Torque [Nm]</td>
</tr>
<tr>
<td>M, m</td>
<td>Mass [kg]</td>
</tr>
<tr>
<td>W</td>
<td>Axle normal load [N], Laden vehicle weight [N]</td>
</tr>
<tr>
<td>f_r</td>
<td>Rolling resistance coefficient [-]</td>
</tr>
<tr>
<td>F</td>
<td>General notation for force [N]</td>
</tr>
<tr>
<td>N</td>
<td>Vertical force [N]</td>
</tr>
<tr>
<td>C_D</td>
<td>Drag coefficient [-]</td>
</tr>
<tr>
<td>A</td>
<td>Frontal area [m^2]</td>
</tr>
<tr>
<td>ρ</td>
<td>Air density [kg/m^3]</td>
</tr>
<tr>
<td>J, I_{CG}, I</td>
<td>Mass moment of inertia [kg.m^2]</td>
</tr>
<tr>
<td>δ</td>
<td>Road wheel steering angle [rad]</td>
</tr>
<tr>
<td>θ</td>
<td>Inclination angle (either for road or for CG test) [rad], oscillation angle for the pendulum [rad]</td>
</tr>
<tr>
<td>φ</td>
<td>Bank angle of the track [rad]</td>
</tr>
<tr>
<td>C_α</td>
<td>Cornering stiffness [N/°]</td>
</tr>
<tr>
<td>α</td>
<td>Slip angle of tyres [°]</td>
</tr>
<tr>
<td>s</td>
<td>Longitudinal slip [-]</td>
</tr>
<tr>
<td>V</td>
<td>Longitudinal speed [m/s] or [km/h] if stated explicitly</td>
</tr>
<tr>
<td>a</td>
<td>Longitudinal acceleration [m/s^2] or [(km/h)/s] if stated explicitly</td>
</tr>
<tr>
<td>ω</td>
<td>Rotational speed [rad/s]</td>
</tr>
<tr>
<td>Ω</td>
<td>Yaw rate [rad/s]</td>
</tr>
<tr>
<td>R</td>
<td>Corner radius [m]</td>
</tr>
<tr>
<td>L</td>
<td>Wheelbase [m]</td>
</tr>
<tr>
<td>b</td>
<td>Track width [m]</td>
</tr>
<tr>
<td>l_f</td>
<td>Distance from CG to the ground contact point of the front wheels [m]</td>
</tr>
<tr>
<td>l_r</td>
<td>Distance from CG to the ground contact point of the rear wheel [m]</td>
</tr>
<tr>
<td>h</td>
<td>CG distance from the ground for Vera [m] or [mm] if stated explicitly, offset distance between the geometric centre of the wheels and CG position for the pendulum when looked from the side [m]</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration [m/s^2]</td>
</tr>
<tr>
<td>P</td>
<td>Tyre inflation pressure [bar], Power [W]</td>
</tr>
<tr>
<td>R_l, r, R</td>
<td>Loaded rolling radius of the tyres</td>
</tr>
<tr>
<td>CG</td>
<td>Centre of gravity</td>
</tr>
<tr>
<td>r</td>
<td>Position vector</td>
</tr>
<tr>
<td>OWC</td>
<td>One way clutch moment [Nm], One way clutch energy loss [W]</td>
</tr>
<tr>
<td>B</td>
<td>Bearing resistance moment [Nm]</td>
</tr>
<tr>
<td>C</td>
<td>Rotational aerodynamic coefficient for the wheels as defined in the text [Nm/s^2]</td>
</tr>
<tr>
<td>D</td>
<td>Rotational aerodynamic resistance moment for the wheels [Nm]</td>
</tr>
<tr>
<td>S</td>
<td>Moment lever arm [m], drop distance [m]</td>
</tr>
<tr>
<td>t</td>
<td>Elapsed time [s]</td>
</tr>
<tr>
<td>p</td>
<td>Vertical distance between the centres of front and rear wheels when front axle is raised [mm]</td>
</tr>
<tr>
<td>α</td>
<td>Rotational acceleration [rad/s^2]</td>
</tr>
</tbody>
</table>
Subscripts

f : Front axle
r : Rear axle, rear wheel
fl : Front left wheel
fr : Front right wheel
wheel : One wheel, effect acting on the wheel
engine : Engine output
OWC : One way clutch (mainly for resistance moment)
H : Hub
T : Traction, friction (force)
roll : Rolling resistance (usually for the rolling res. moment)
bearing : Bearing resistance
res : Resistance (sum of)
aero : Aerodynamic
tot : total (usually referring to the total mass of Vera)
av : Average
start : Start (speed)
stop : Stop (speed)
x, y, z : Effect on these axes
added : Additional (mass)

Superscript

* : New quantity after front axle is raised
References

Books:


Benchmarking others teams:

- European Shell Eco-Marathon Lausitz 2009 Ranking
  - http://la-joliverie.com/Microjoule-Polyjoule.91.0.html
  - http://gdarves.free.fr/site/microjoule.html

Vera data:

- “Implementing advanced CAE tools in automotive engineering education at Chalmers University of Technology “, by Lasse Christoffersen, Christoffer Landström and Lennart Löfdahl, Chalmers University of Technology/Applied Mechanics Department, Sweden.

- Eco team map network drive called "ecomarathon" on file.chalmers.se

Vortex generation:

- “Automated pre-processing for high quality multiple variant CFD models of a city-class car”, by Evangelos Skaperdas, Christos Kolovos, Beta CAE Systems S.A., Greece.
Bearings:

- Bearing frictional moment calculation: [http://www.skf.com/skf/productcatalogue/calculationFilter?print=true&action=Calc5&calc_extraInfo=false&lang=en&DW2=1.19&newlink=&H=0&next=ok&reloading=false&myEHL=0.05&Fa=&Fr=&info=Calc5&seals=2&windowName=fhvMkP6MTZwT1GS5LOY14Y_1252878951358_Calc5&ny=&f01Radio=0&n=&calcform=form1&prodid=101081802&mainCatalogue=1&lang=en](http://www.skf.com/skf/productcatalogue/calculationFilter?print=true&action=Calc5&calc_extraInfo=false&lang=en&DW2=1.19&newlink=&H=0&next=ok&reloading=false&myEHL=0.05&Fa=&Fr=&info=Calc5&seals=2&windowName=fhvMkP6MTZwT1GS5LOY14Y_1252878951358_Calc5&ny=&f01Radio=0&n=&calcform=form1&prodid=101081802&mainCatalogue=1&lang=en)
- Bearing test suggestion: “Laboratory work notes for the frictional losses in bearings”, Nizhny Novgorod State Technical University, Ulianov A.A., Zhelandovskij V.E., Chumakov O.I., 2006

Tires:

- Standard SAE J2452 defined by the SAE to define the rolling resistance of a tire.

Chain:

- “Validation of a mathematical model for road cycling power”, by James C. Martin, Douglas L. Milliken, John E. Cobb, Kevin L. McFadden and Andrew R. Coggan.
- “Measuring drive chain efficiency”, by Angus Cameron.
- “Effects of frictional loss on bicycle chain drive efficiency”, by James B. Spicer, Cristopher Richardson, Michael Ehrlich and Johanna Bernstein, Johns Hopkins University, Baltimore, and by Masahiko Fukuda and Masao Terada, Shimano Inc., Product Engineering Division, Osaka.
- “Efficiency measurements of bicycle transmissions, a never-ending story?”, by Bernhard Rohloff and Peter Greb.

Tracks:

- [http://www.eurospeedway.de/](http://www.eurospeedway.de/)